

CHAPTER 6 / 6A

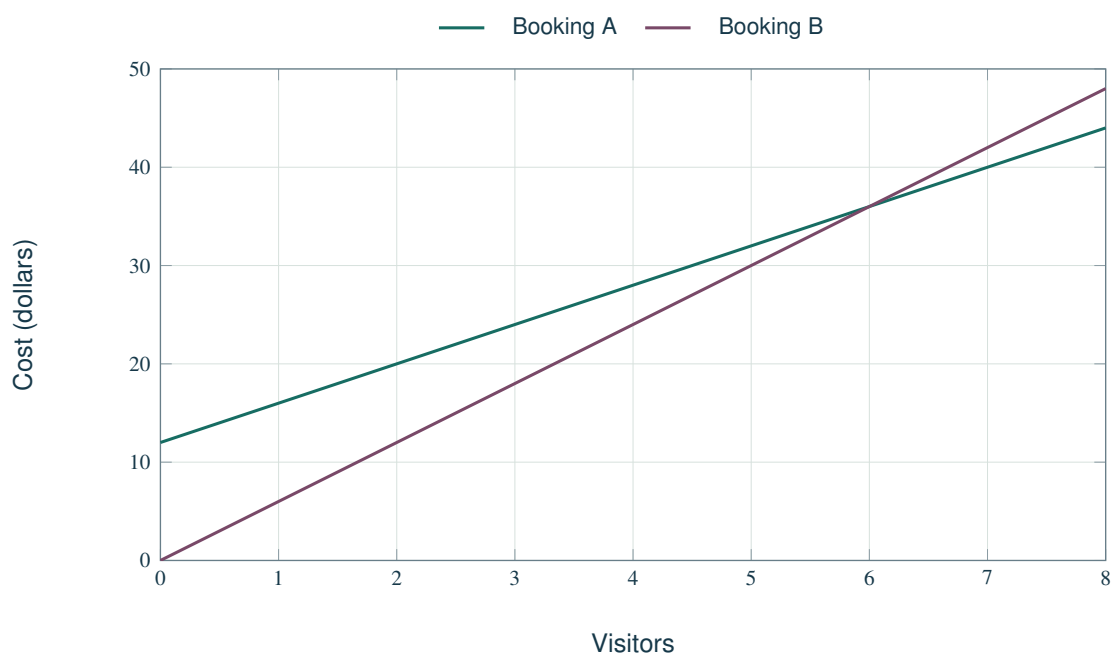
Interpreting straight-line graphs

Name: _____ Class: _____ Date: _____

Use graphs to make decisions. Read both axes carefully, distinguish starting values from changes, and decide when a straight-line forecast is reasonable.

What you will learn

- Read values and intercepts using the displayed scale.
- Compare two straight-line relationships in context.
- Use interpolation and cautious extrapolation to make decisions.

See the mathematics

Learning map: compare starting values, readings at one input, and where two lines meet.

Key ideas

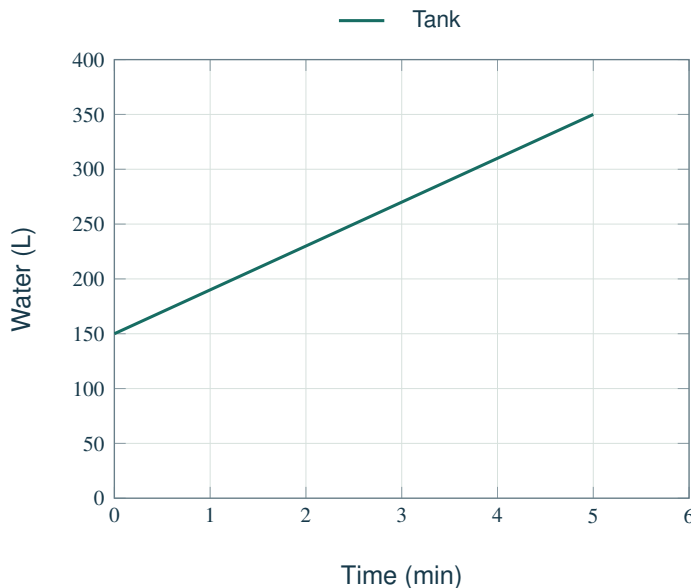
- The horizontal and vertical axes represent different quantities and units.
- An intercept records a value when the other variable is zero.
- A straight line represents a constant change over equal input intervals.
- Interpolation uses the recorded interval; extrapolation extends a trend beyond it.

Pages 1–8: student resource. Pages 9–10: worked answers.

Worked examples / connect the calculation to the graph

1. A stored-water forecast

A tank graph passes through $(0, 150)$ and $(5, 350)$, where time is in minutes and water is in litres. Find the volume at 3 minutes.



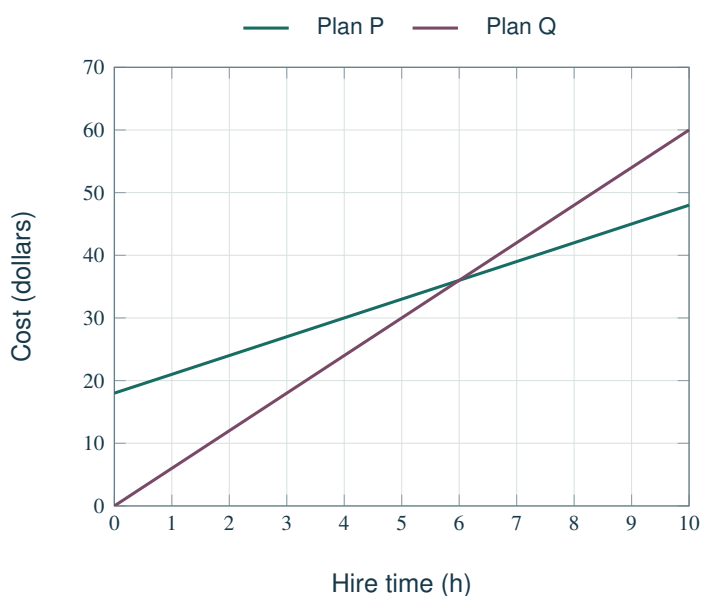
1. The volume increases by $350 - 150 = 200$ L in 5 min, or 40 L each minute.
2. At 3 min, the tank holds its initial 150 L plus 3×40 L.

Result
270 L.

Worked example: read the volume within the measured filling interval.

2. Two hire plans

Plan P costs $C = 18 + 3h$. Plan Q costs $C = 6h$. Find when the costs are equal and the cheaper plan for 8 hours.



1. Set $18 + 3h = 6h$, giving $h = 6$.
2. At $h = 8$, P costs \$42 and Q costs \$48.

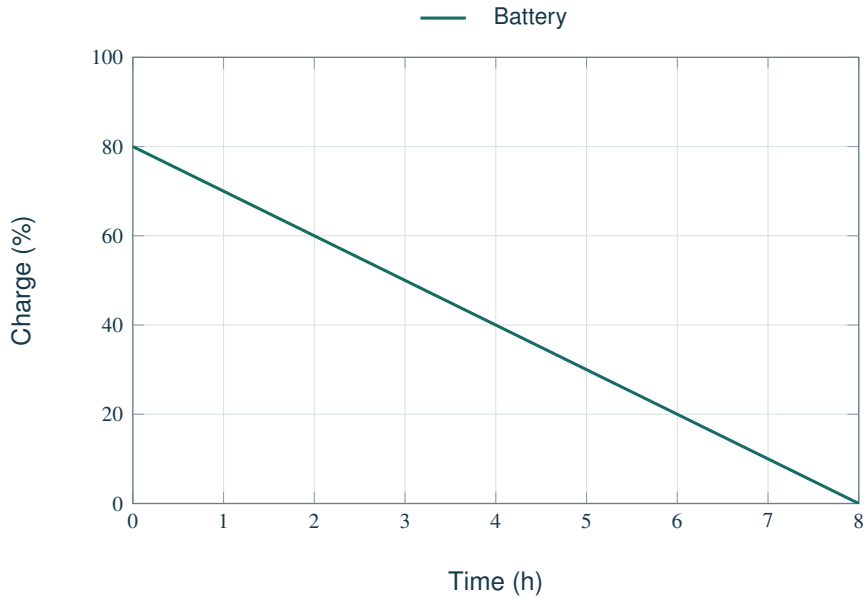
Result
Equal at 6 hours; P is cheaper for 8 hours.

Worked example: compare costs at the same hire time.

Read the graph

Use the graph to record your thinking. Show calculations and include units where needed.

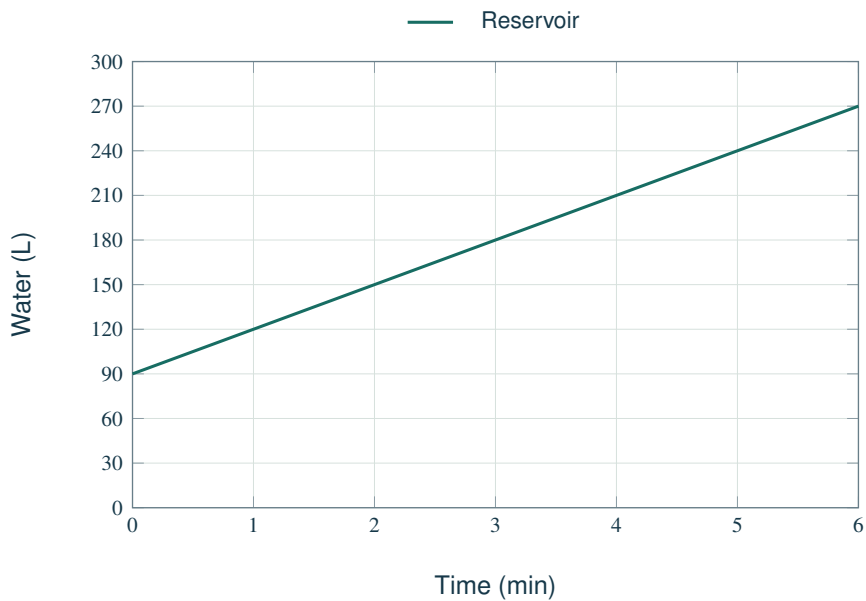
1. The graph shows a battery’s charge. Read its starting charge, its charge after 4 hours, and the first time it reaches 20%.



WORKING / DECISION

Read the quantities, scales and labelled line before answering.

2. A reservoir graph passes through (0, 90), (2, 150) and (6, 270), with time in minutes and volume in litres. What does (0, 90) mean? How much water is present at 4 minutes?



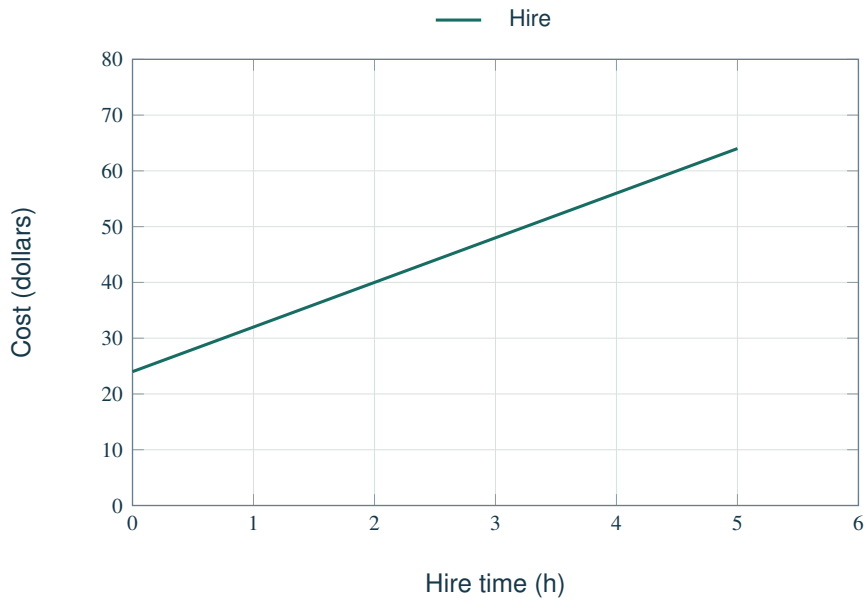
WORKING / DECISION

Read the initial water and the volume at the requested time.

Read the graph

Use the graph to record your thinking. Show calculations and include units where needed.

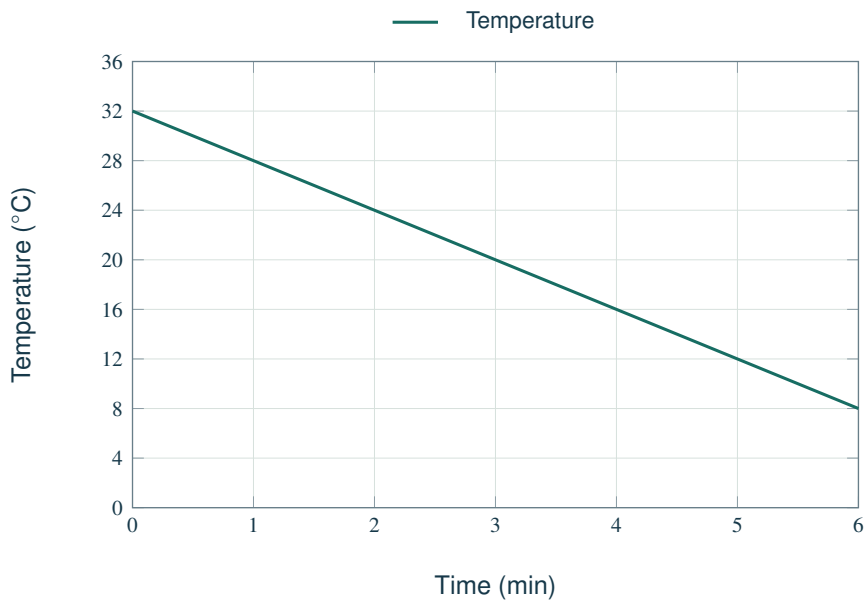
3. An equipment-hire graph has points (0, 24), (2, 40) and (5, 64), where x is hours and y is dollars. Find the booking fee and the charge for 3 hours.



WORKING / DECISION

Use the cost graph to distinguish the booking charge from the time charge.

4. A machine's temperature graph falls from (0, 32) to (6, 8), with time in minutes. Read the time when it is 20°C. Explain what the falling line tells you.



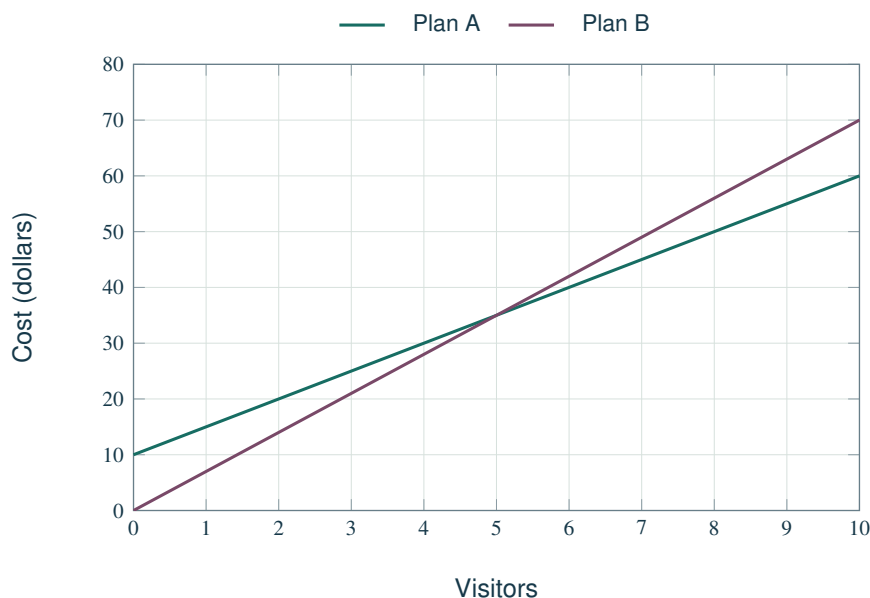
WORKING / DECISION

Read the quantities, scales and labelled line before answering.

Compare and interpret

Use the graph to record your thinking. Show calculations and include units where needed.

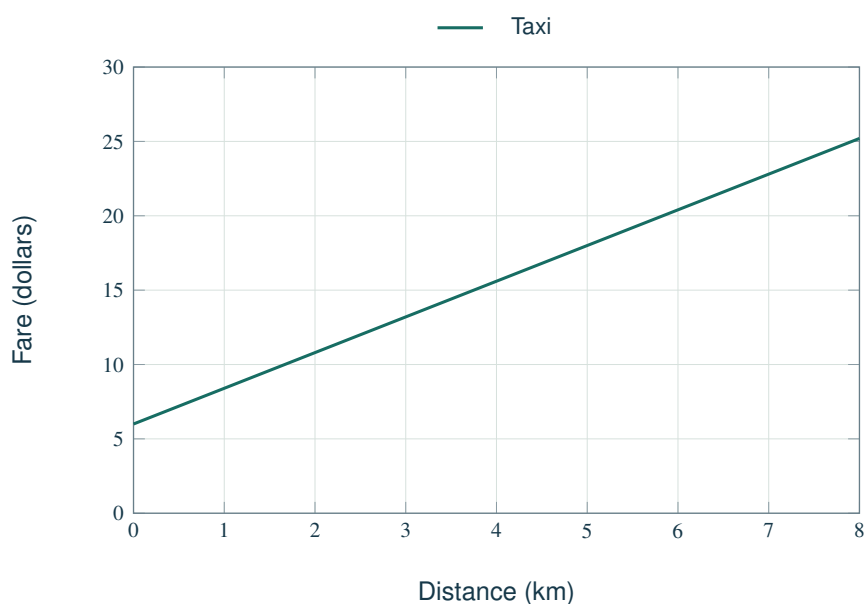
5. Two admission plans are $C = 10 + 5n$ and $C = 7n$, where n is visitors. Find the equal-price group size and choose the cheaper plan for 9 visitors.



WORKING / DECISION

Compare both booking plans at the same group size.

6. A taxi fare is $F = 6 + 2.4d$ dollars for d kilometres. A passenger says the point $(0, 6)$ means 6 km are free. Correct the statement and find the fare for 5 km.



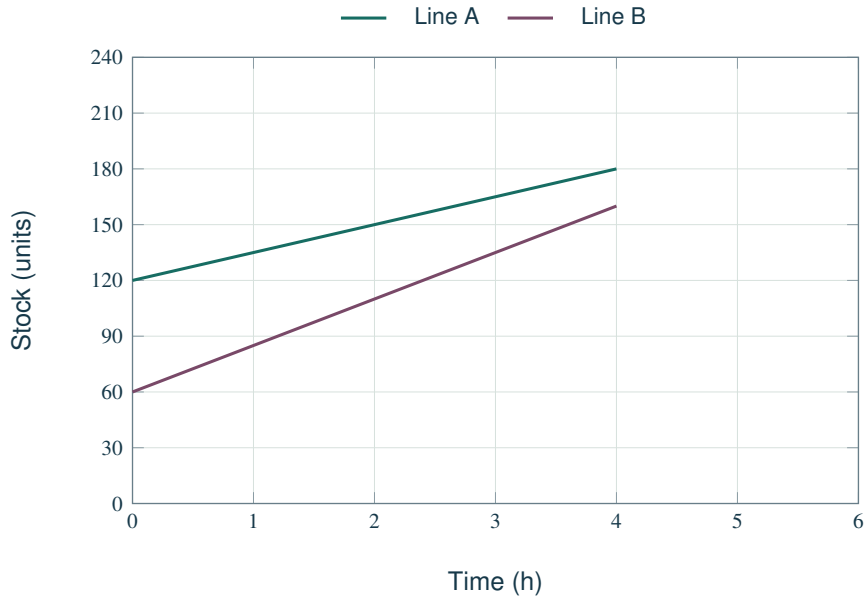
WORKING / DECISION

Interpret where the fare graph meets the vertical axis.

Compare and interpret

Use the graph to record your thinking. Show calculations and include units where needed.

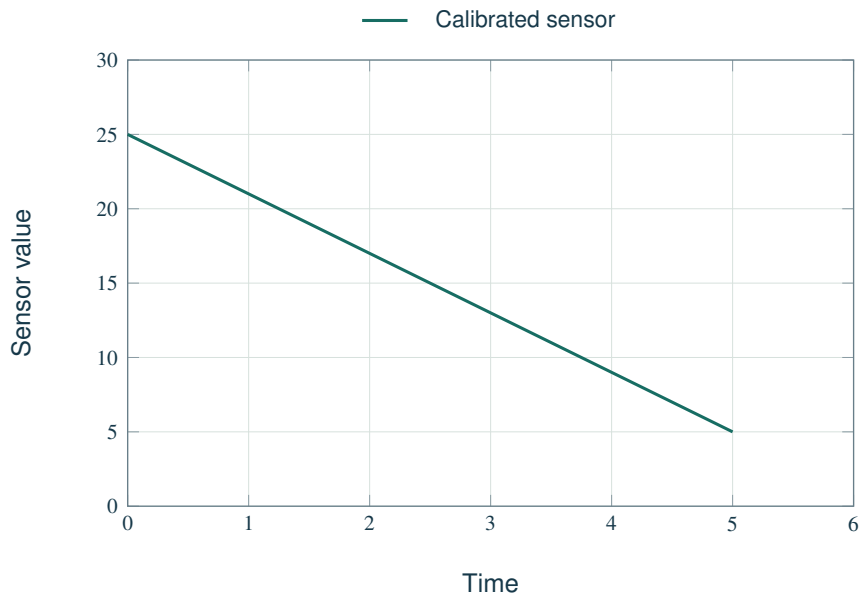
7. Line A records inventory at (0, 120) and (4, 180). Line B records (0, 60) and (4, 160). Which line has more stock at 4 hours, and which grows faster?



WORKING / DECISION

Compare starting stock, later stock and changes over the same interval.

8. A calibrated sensor follows the straight line through (0, 25) and (5, 5). Its log reports A: (1, 21), B: (3, 15), C: (4, 9). Identify and correct the faulty record.



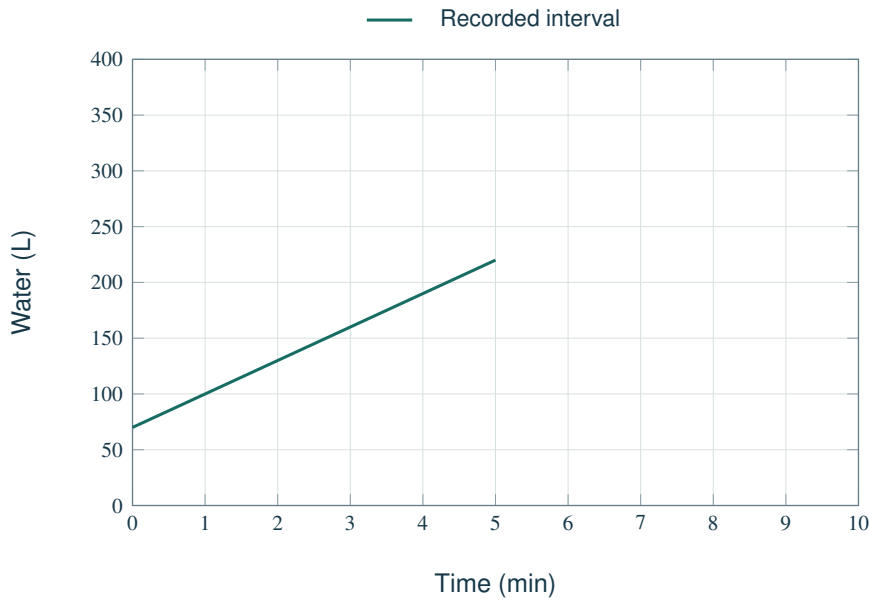
WORKING / DECISION

Compare the three records in the question with the calibrated line.

Forecast and decide

Use the graph to record your thinking. Show calculations and include units where needed.

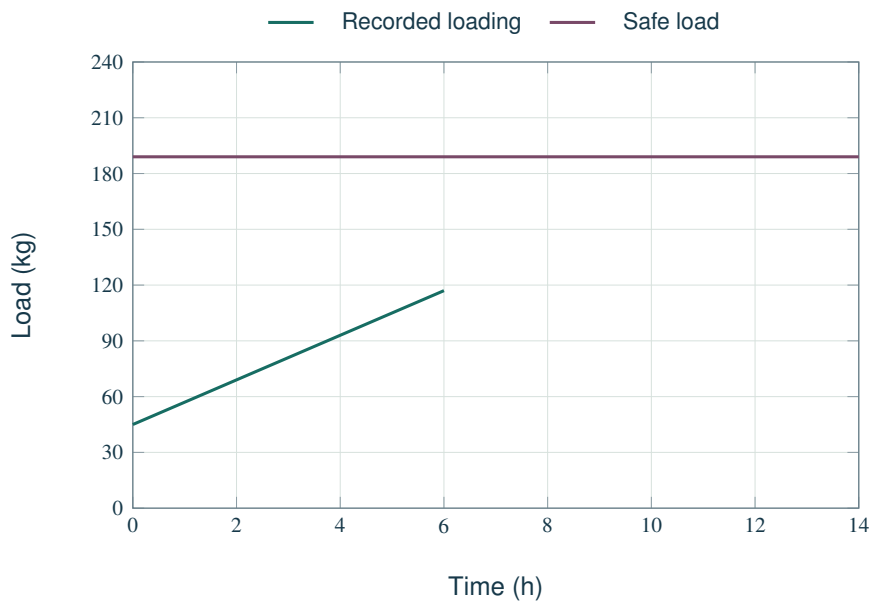
9. A pump's recorded graph runs from (0, 70) to (5, 220), with minutes and litres. Forecast the volume at 9 minutes. Is this interpolation or extrapolation?



WORKING / DECISION

Continue the recorded trend yourself to make the requested forecast.

10. A storage bin contains 45 kg initially and gains 12 kg each hour. Its maximum safe load is 189 kg. Find the shutoff time and name one condition your forecast assumes.



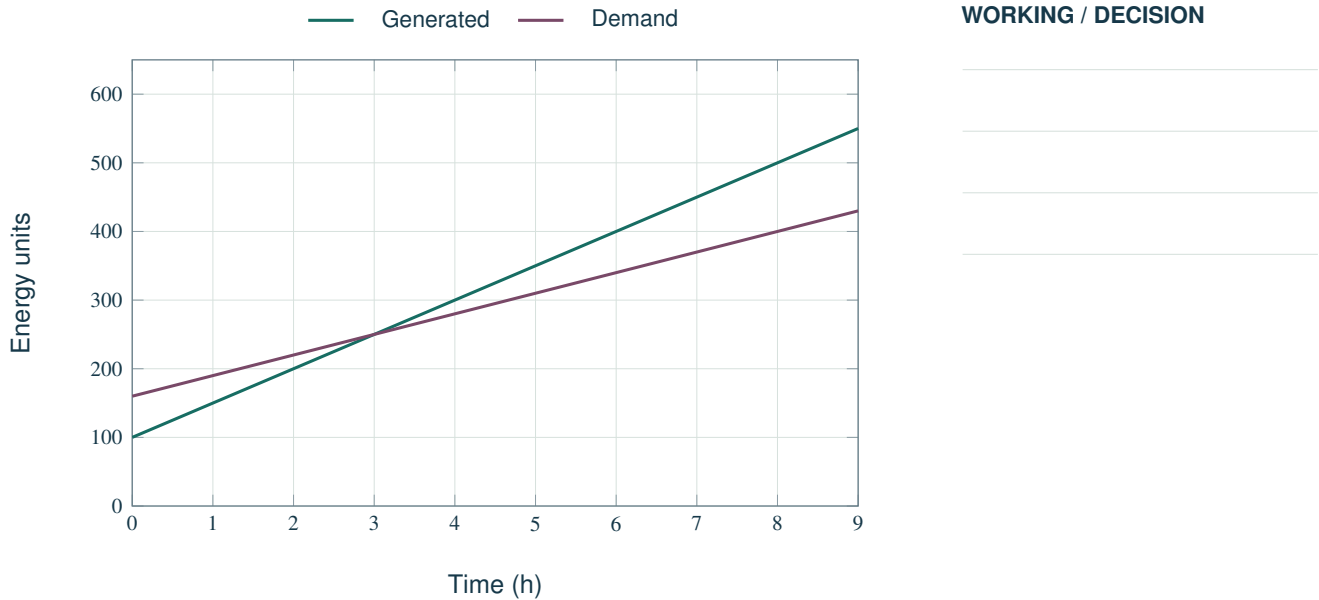
WORKING / DECISION

The safe-load line is supplied; determine where your continued loading plan reaches it.

Forecast and decide

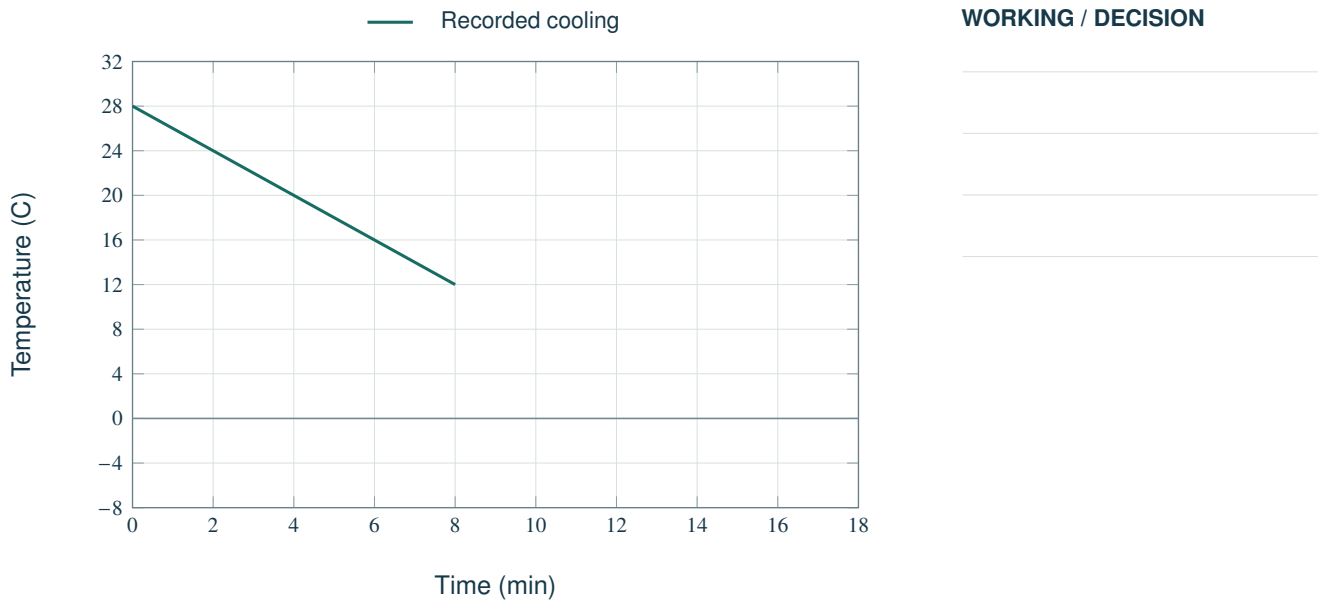
Use the graph to record your thinking. Show calculations and include units where needed.

11. Generated energy is $G = 50t + 100$ and demand is $D = 30t + 160$, in the same units. Find when they balance and the earliest whole hour with a reserve of at least 80 units.



Read both lines at one time to compare the amount generated with the amount required.

12. A cooling line fits $T = 28 - 2t$ from $t = 0$ to $t = 8$ min. Find T at 6 min and predict T at 16 min. Give a reason the later prediction may not describe the real machine.



Only the recorded interval is drawn. Add your own later prediction and discuss its limits.

6A / Worked answers

Teacher appendix. Equivalent exact forms are accepted. Use the reasoning to check student work.

1. 80%; 40%; 6 hours.

- The line starts at $(0, 80)$.
 - At $x = 4$ the vertical value is 40; the $y = 20$ level meets the line at $x = 6$.
-

2. Initially 90 L; 210 L at 4 min.

- The reservoir already contains 90 L when the clock starts.
 - Volume grows by 30 L/min, so $90 + 4 \times 30 = 210$.
-

3. \$24 booking fee; \$48 for 3 hours.

- At zero hours the cost is \$24.
 - The line rises \$8 per hour; $24 + 3 \times 8 = 48$.
-

4. 3 minutes; the machine cools as time increases.

- The temperature decreases by 4°C each minute.
 - A drop from 32°C to 20°C is 12°C , requiring $\frac{12}{4} = 3$ min.
-

5. Equal at 5 visitors; the first plan is cheaper at \$55.

- $10 + 5n = 7n$ gives $n = 5$.
 - At $n = 9$ the costs are \$55 and \$63.
-

6. It means a \$6 starting charge; the 5 km fare is \$18.

- When $d = 0$, the fare is already \$6.
 - $F = 6 + 2.4 \times 5 = 18$.
-

6A / Worked answers

Teacher appendix. Equivalent exact forms are accepted. Use the reasoning to check student work.

7. A has more stock; B grows faster.

- At 4 h, A has 180 and B has 160.
 - A gains 15 units/h while B gains 25 units/h.
-

8. B is faulty; it should be (3, 13).

- The value falls by 4 each time unit.
 - At $x = 3$, $y = 25 - 4 \times 3 = 13$; the other records fit.
-

9. 340 L; extrapolation.

- Rate = $(220 - 70)/5 = 30$ L/min.
 - Volume at 9 min is $70 + 30 \times 9 = 340$ L, outside the recorded interval.
-

10. 12 hours; the loading rate must remain 12 kg/h.

- The bin can gain $189 - 45 = 144$ kg.
 - $\frac{144}{12} = 12$ h.
-

11. Balance at 3 h; reserve target first met at 7 h.

- $50t + 100 = 30t + 160$ gives $t = 3$.
 - Reserve = $G - D = 20t - 60$; $20t - 60 \geq 80$ gives $t \geq 7$.
-

12. 16°C; predicted -4°C; cooling may slow or stop near room temperature.

- At 6 min, $T = 28 - 12 = 16$.
 - At 16 min, $T = 28 - 32 = -4$; this extrapolates beyond observed data.
-

CHAPTER 6 / 6B

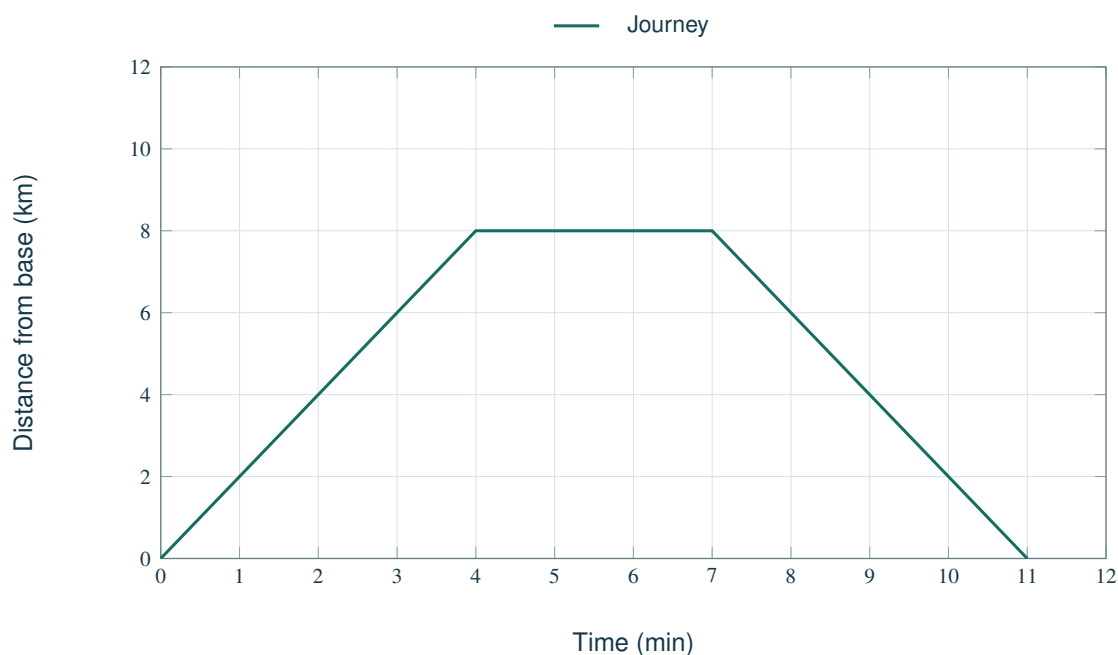
Distance–time graphs

Name: _____ Class: _____ Date: _____

Tell the story of a journey from its graph. All journeys in this worksheet take place along one straight route. Distinguish distance from base from total distance travelled, and use time intervals to compare motion.

What you will learn

- Interpret sloping and horizontal journey segments.
- Calculate travel times, speeds and total distance.
- Construct and compare complete journey plans.

See the mathematics

Learning map: follow an outward trip, a stop and a return along one straight road.

Key ideas

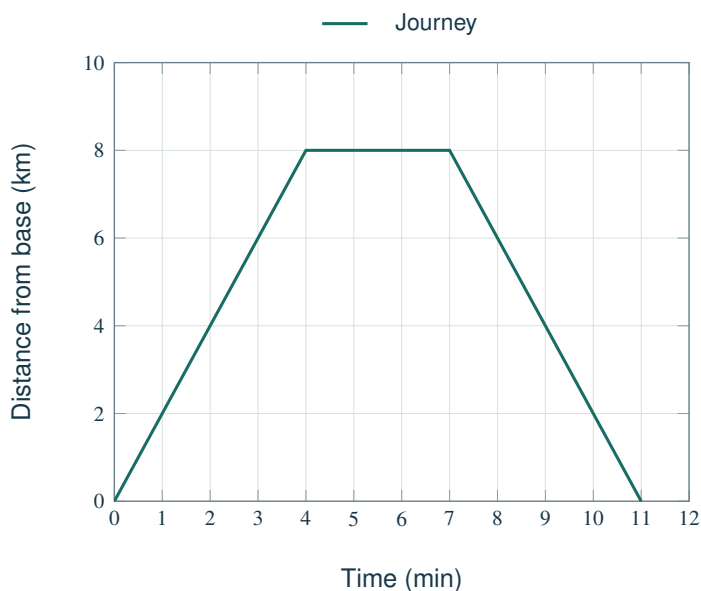
- Read the vertical-axis label: distance from base can decrease on a return trip.
- For motion along one straight route, a flat distance-from-base segment means the traveller is stationary.
- Gradient gives speed magnitude; a downward distance-from-base segment indicates return.
- Total distance is the sum of the magnitudes of each moving segment's distance change.

Pages 1–8: student resource. Pages 9–10: worked answers.

Worked examples / connect the calculation to the graph

1. Out, wait and return

A tracker records $(0, 0)$, $(4, 8)$, $(7, 8)$, $(11, 0)$, in minutes and kilometres from base. Find the stop duration and total distance travelled.



1. The horizontal section lasts $7 - 4 = 3$ min.
2. The vehicle travels 8 km outward and 8 km back.

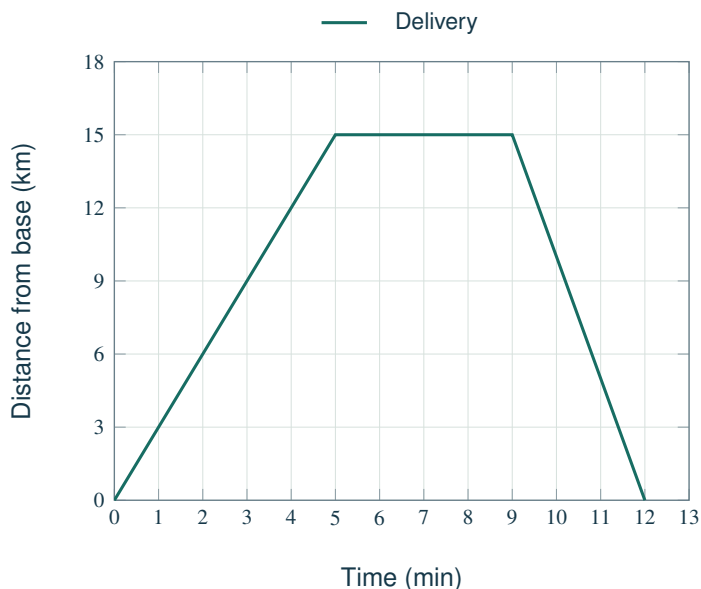
Result

Stop: 3 min. Total distance: 16 km.

Worked example: a stop and return are separate parts of the full trip.

2. A faster return

A vehicle travels 15 km out at 3 km/min, waits 4 minutes and returns at 5 km/min. Find the total time.



1. Outward time = $\frac{15}{3} = 5$ min; return time = $\frac{15}{5} = 3$ min.
2. Add the 4-minute stop: $5 + 4 + 3 = 12$ min.

Result

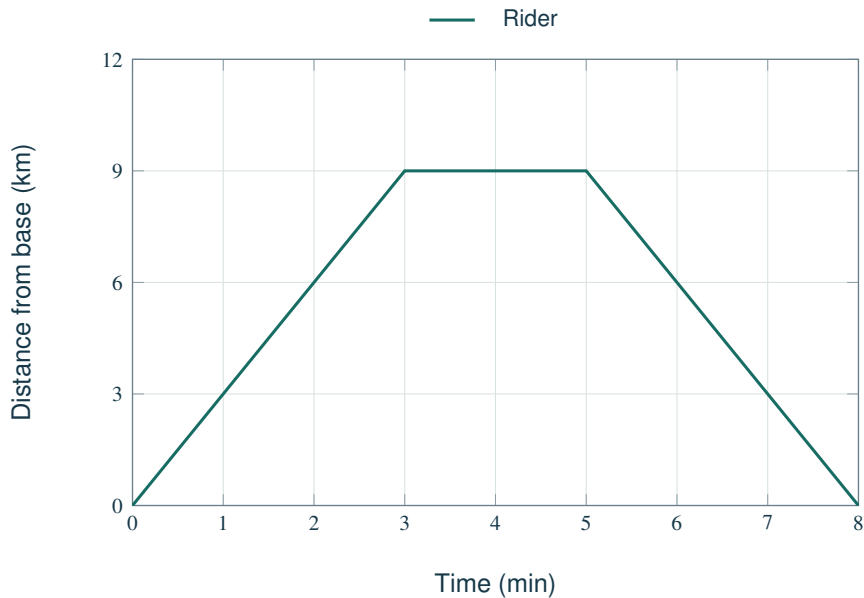
12 minutes.

Worked example: the return trip is faster than the outward trip.

Read a recorded journey

Use the graph to record your thinking. Show calculations and include units where needed.

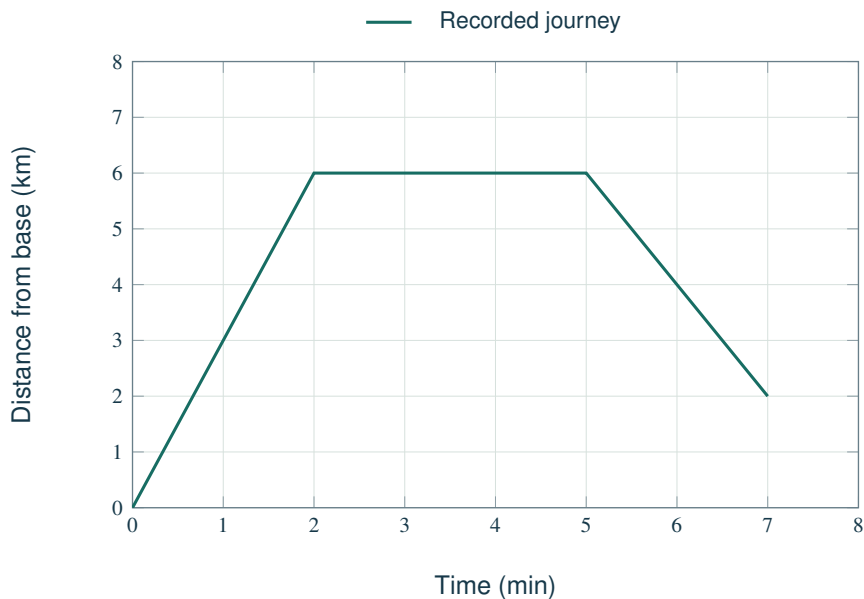
1. Read the journey graph. How far is the rider from base at 2 minutes? When is the rider stationary? When does the rider return to base?



Read the quantities, scales and labelled line before answering.

WORKING / DECISION

2. A journey is recorded as $(0,0)$, $(2,6)$, $(5,6)$, $(7,2)$, with time in minutes and distance from base in kilometres. Find the outward speed and the stop duration.



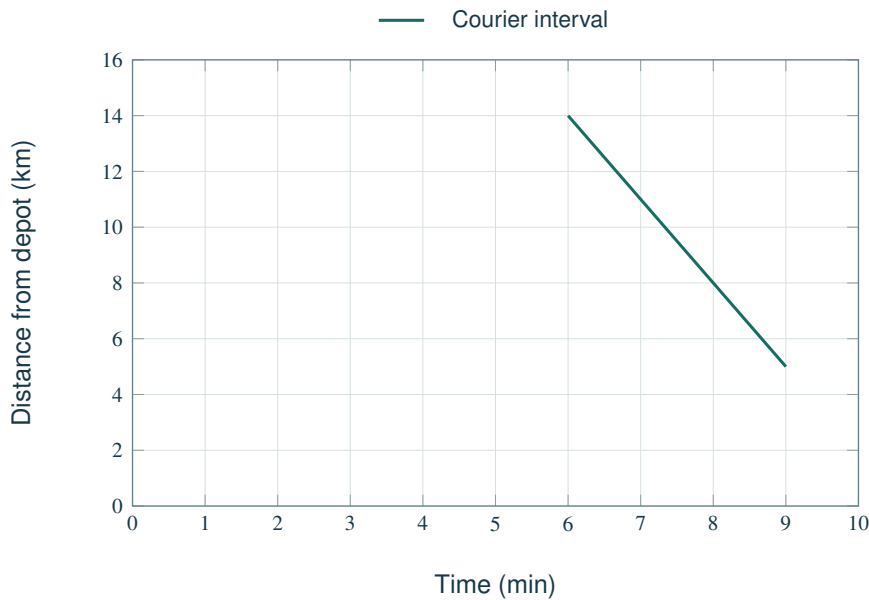
Use the joined segments to distinguish travel from waiting.

WORKING / DECISION

Read a recorded journey

Use the graph to record your thinking. Show calculations and include units where needed.

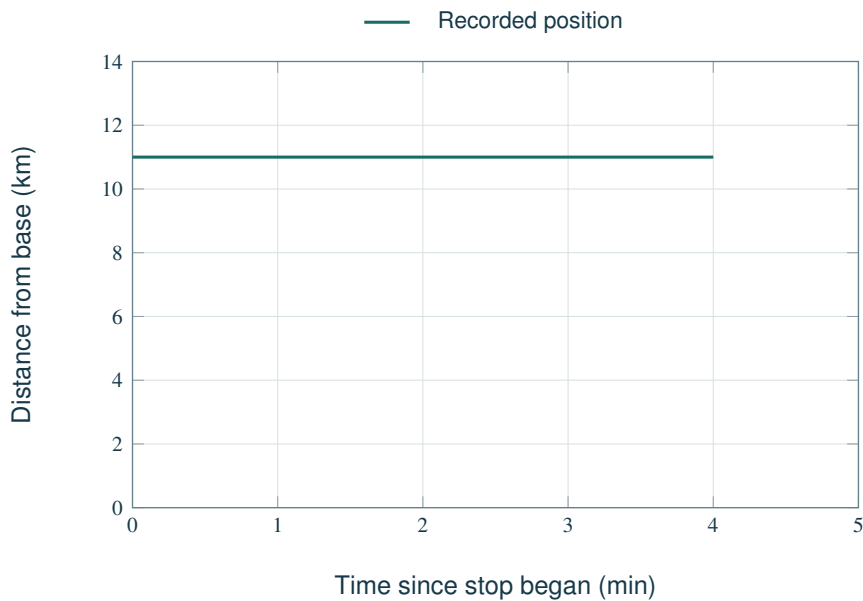
3. A courier's distance from the depot changes from 14 km at 6 minutes to 5 km at 9 minutes. Is the courier heading toward or away from the depot? Find the speed.



WORKING / DECISION

Only the measured interval is shown. Interpret its direction and rate.

4. A graph is flat at a distance of 11 km from base for 4 minutes. A student says the vehicle travels 11 km in those 4 minutes. Explain the error.



WORKING / DECISION

Describe what changes, and what stays fixed, during this interval.

Reconstruct and compare

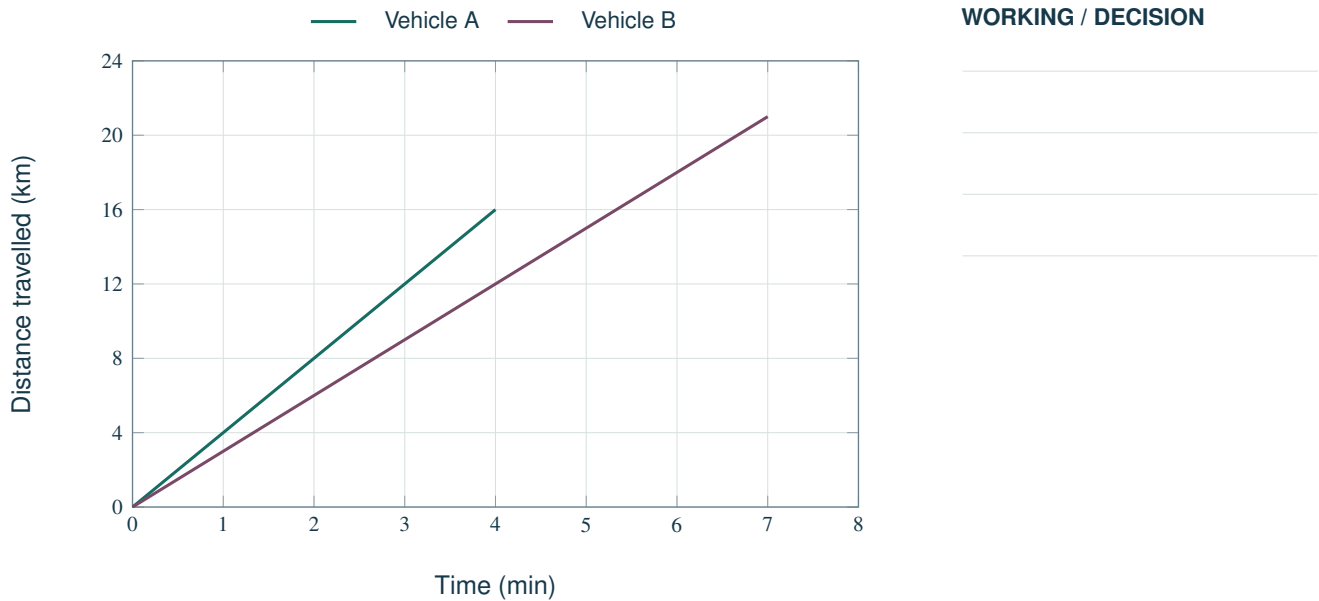
Use the graph to record your thinking. Show calculations and include units where needed.

5. A route goes from base to 10 km away, returns to the 4 km mark, then goes outward to the 7 km mark. Find total distance travelled and final distance from base.



Checkpoints show travel order only. Equal horizontal spacing does not imply equal travel time.

6. Vehicle A travels 16 km in 4 min. Vehicle B travels 21 km in 7 min. Which has the steeper distance–time graph when the scales are identical?

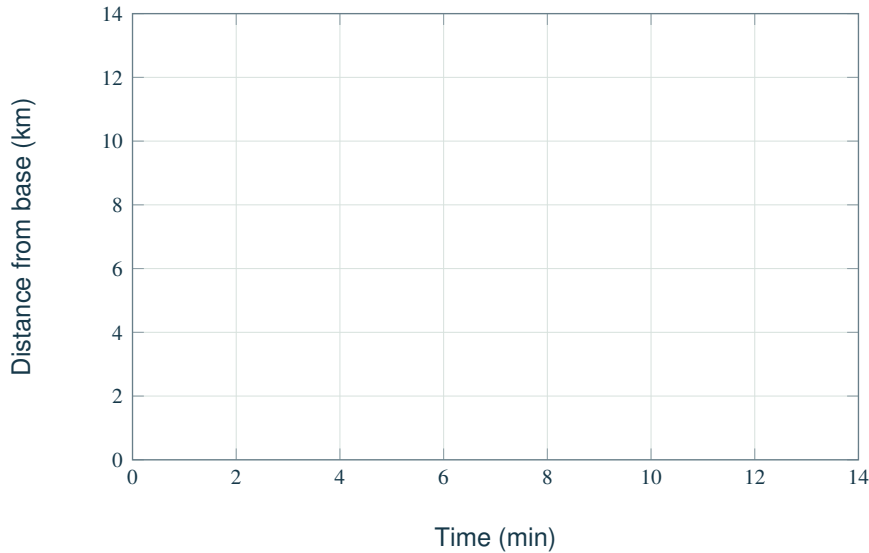


Compare the vehicles on the same time and distance scales.

Reconstruct and compare

Use the graph to record your thinking. Show calculations and include units where needed.

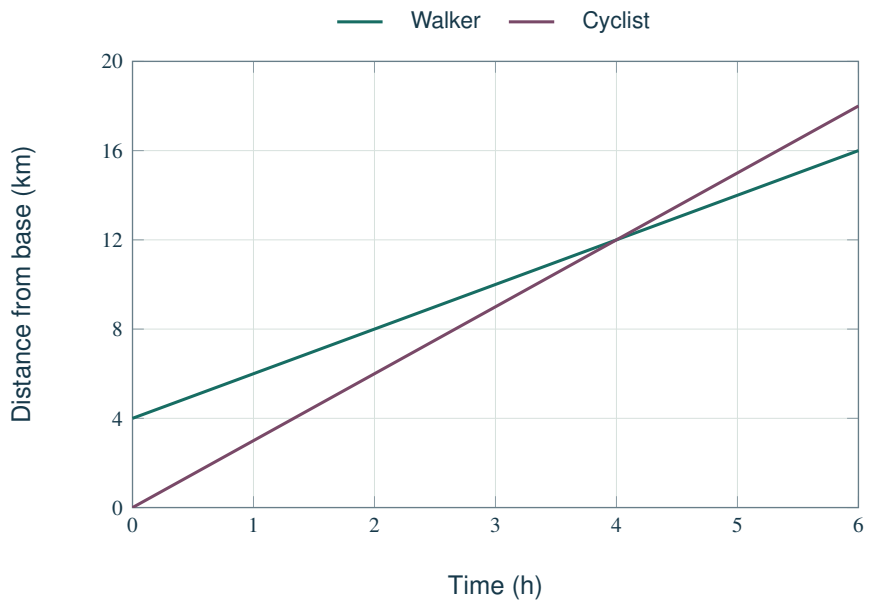
7. Sketch a distance-from-base graph for a 12 km outward trip in 4 min, a 3 min stop, and a return to base in 6 min. Give the coordinates of each change of activity.



Use these labelled axes to construct and check the requested graph.

WORKING / DECISION

8. The graph shows a walker and a cyclist on the same straight path, with time measured in hours. Find their meeting time and distance from base. Which starts ahead?



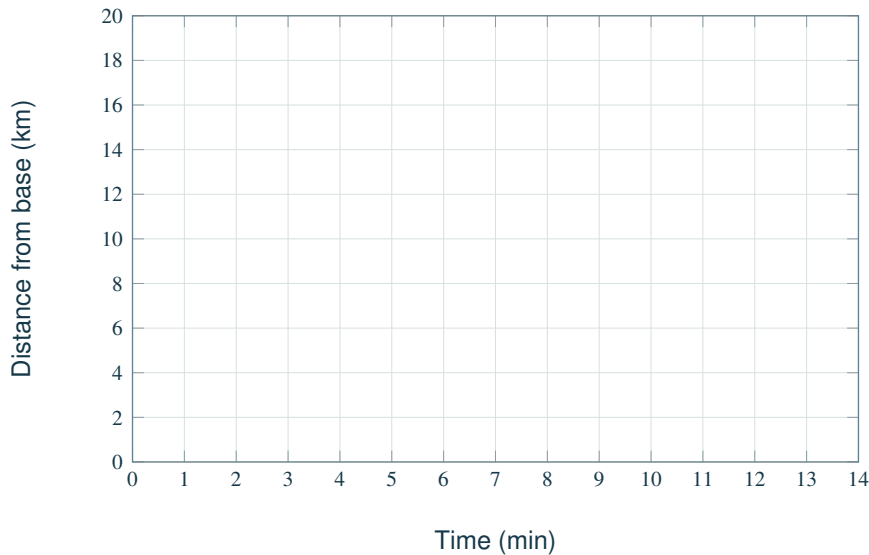
Read the quantities, scales and labelled line before answering.

WORKING / DECISION

Plan the whole journey

Use the graph to record your thinking. Show calculations and include units where needed.

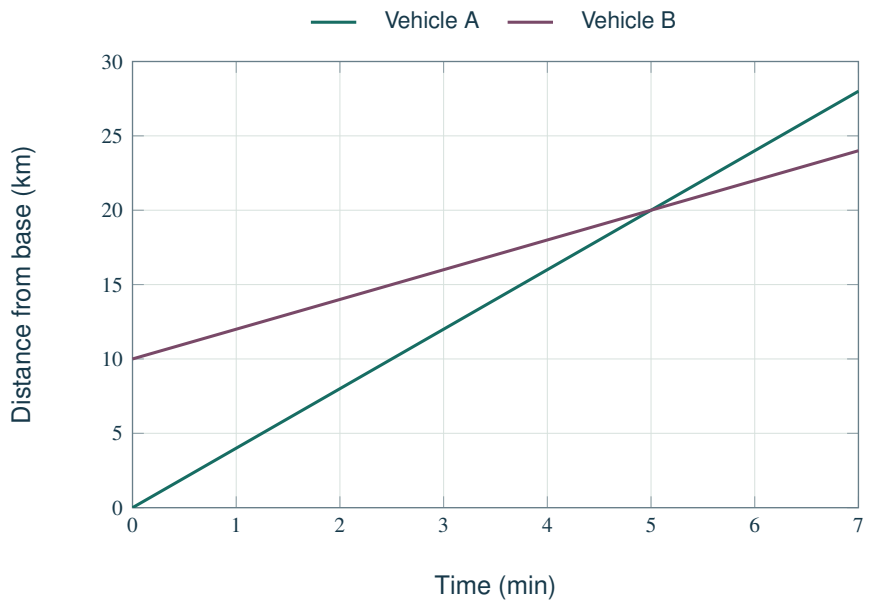
9. A van travels 18 km out at 3 km/min, stops for 2 min and returns at 6 km/min. Will it finish within 12 minutes? Show the complete time.



WORKING / DECISION

Draw the complete outward journey, stop and return before checking the deadline.

10. Two vehicles follow $dA = 4t$ and $dB = 2t + 10$, with time in minutes and distance from base in kilometres. Find when and where A catches B.



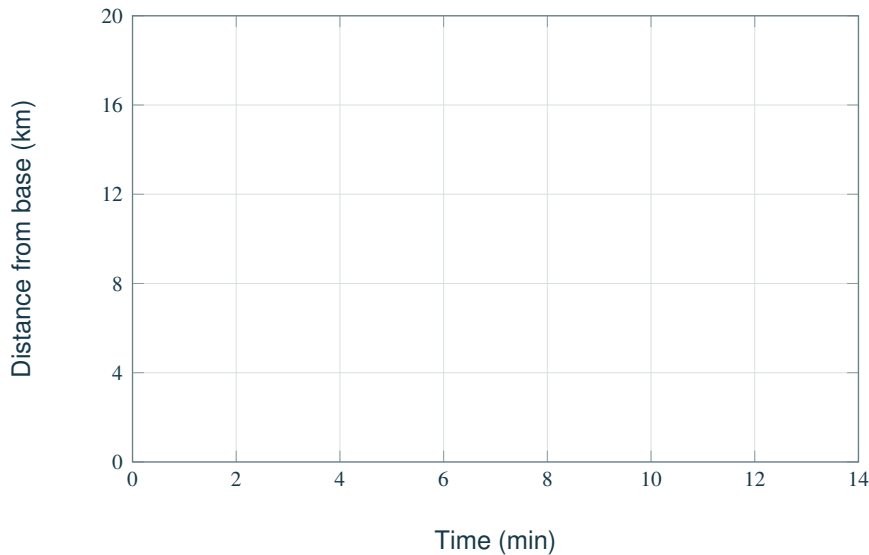
WORKING / DECISION

A handover needs the same position at the same time.

Plan the whole journey

Use the graph to record your thinking. Show calculations and include units where needed.

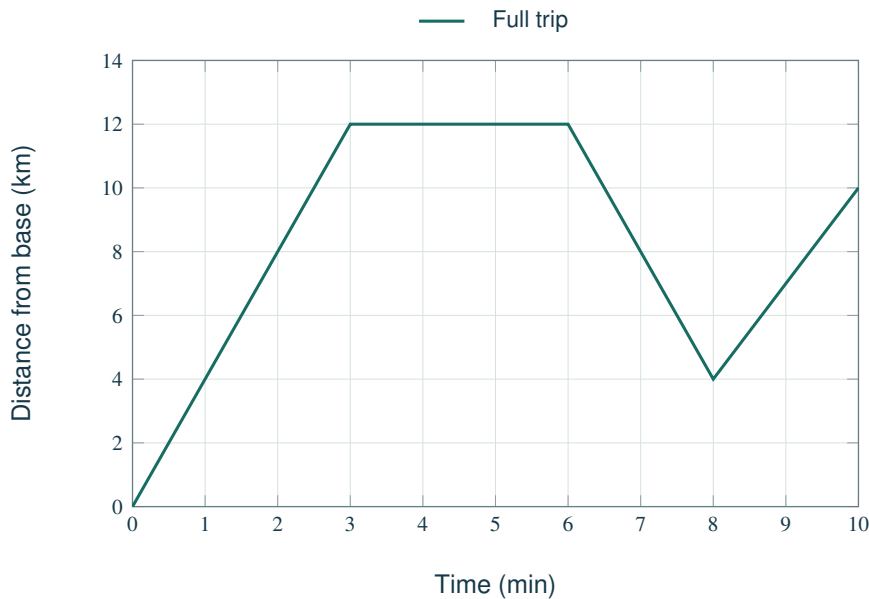
11. A 16 km delivery requires a 4-minute stop and a return to base. Van P costs \$80 and travels 4 km/min. Van Q costs \$115 and travels 8 km/min. The deadline is 12 minutes and the budget is \$100. Choose a van.



WORKING / DECISION

Sketch the plan for each candidate, including the delivery stop. Use the hire costs from the question.

12. A trip has points (0, 0), (3, 12), (6, 12), (8, 4), (10, 10), in minutes and kilometres from base. Find the total distance, total stationary time and greatest speed.



WORKING / DECISION

The axes show distance from base, not an odometer reading.

6B / Worked answers

Teacher appendix. Equivalent exact forms are accepted. Use the reasoning to check student work.

1. 6 km; from 3 to 5 min; returns at 8 min.

- The first segment rises 3 km per minute, so at 2 min the rider is 6 km out.
 - The graph is horizontal between 3 and 5 min, then reaches zero at 8 min.
-

2. 3 km/min; 3-minute stop.

- Outward speed = $\frac{6}{2} = 3$ km/min.
 - The stop lasts from minute 2 to minute 5, or 3 min.
-

3. Toward the depot; 3 km/min.

- Distance from depot decreases by 9 km.
 - Speed magnitude = $\frac{9}{9-6} = 3$ km/min.
-

4. It remains 11 km from base and travels no distance during the flat section.

- The vertical value describes position relative to base.
 - A flat section has zero change in distance from base, so the vehicle is stationary.
-

5. 19 km travelled; 7 km from base.

- Add 10 km out, $10 - 4 = 6$ km back and $7 - 4 = 3$ km out.
 - $10 + 6 + 3 = 19$ km; the final position is 7 km.
-

6. Vehicle A.

- A's speed is $\frac{16}{4} = 4$ km/min.
 - B's speed is $\frac{21}{7} = 3$ km/min, so A's graph is steeper.
-

6B / Worked answers

Teacher appendix. Equivalent exact forms are accepted. Use the reasoning to check student work.

7. (0, 0), (4, 12), (7, 12), (13, 0), joined in order.

- After driving 4 min the vehicle is 12 km away.
 - The stop ends at 7 min; the 6 min return ends at 13 min.
-

8. Meet at 4 hours and 12 km; walker starts 4 km ahead.

- The lines intersect at (4, 12): 4 hours after the cyclist starts, 12 km from base.
 - At time zero the walker is at 4 km while the cyclist is at 0 km.
-

9. Yes; it takes 11 minutes.

- Outward time is $\frac{18}{3} = 6$ min; return time is $\frac{18}{6} = 3$ min.
 - Total = $6 + 2 + 3 = 11$ min.
-

10. 5 minutes; 20 km from base.

- Set $4t = 2t + 10$, so $t = 5$.
 - Substitute into either expression: $d = 20$ km.
-

11. Van P; 12-minute journey costing \$80.

- P takes $\frac{32}{4} + 4 = 12$ min and is within budget.
 - Q takes $\frac{32}{8} + 4 = 8$ min but exceeds the \$100 budget.
-

12. 26 km; 3 min stationary; greatest speed 4 km/min.

- Total distance = $12 + 8 + 6 = 26$ km; the stop lasts $6 - 3 = 3$ min.
 - Moving speeds are $\frac{12}{3} = 4$, $\frac{8}{2} = 4$ and $\frac{6}{2} = 3$ km/min.
-

CHAPTER 6 / 6C

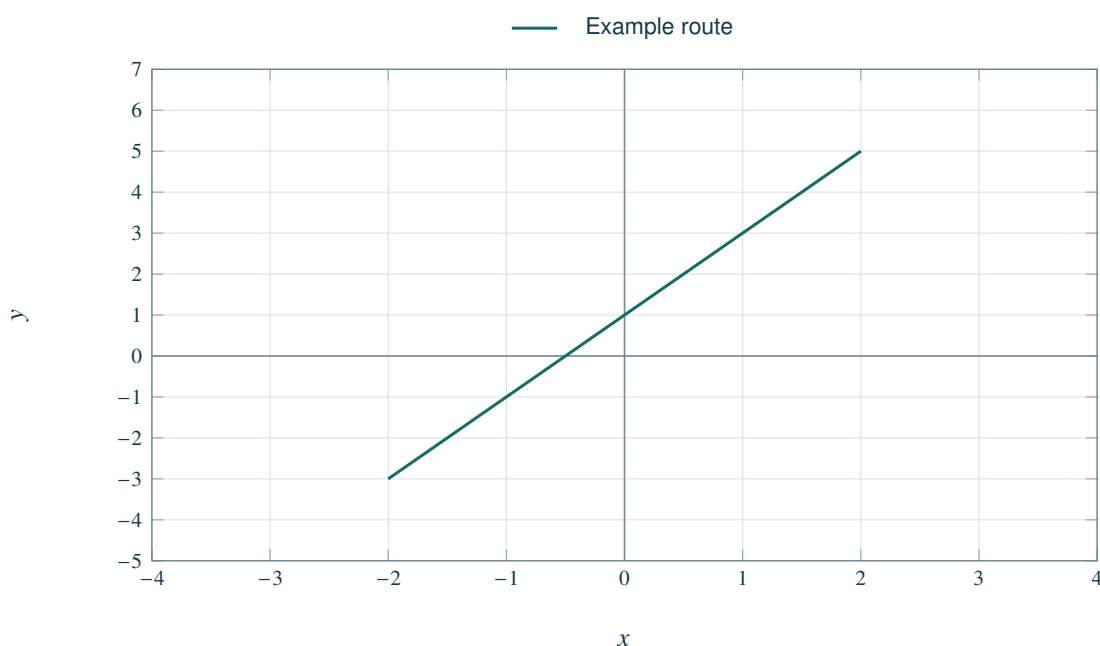
Graphing straight lines

Name: _____ Class: _____ Date: _____

Build straight-line graphs from equations and tables. Work across all four quadrants, check suspect coordinates, and use a graph to find missing values.

What you will learn

- Interpret and plot signed coordinates.
- Build a value table from a linear equation.
- Check whether points belong to a line and use the line to recover missing coordinates.

See the mathematics

Learning map: a table of matching horizontal and vertical coordinates makes a straight route.

Key ideas

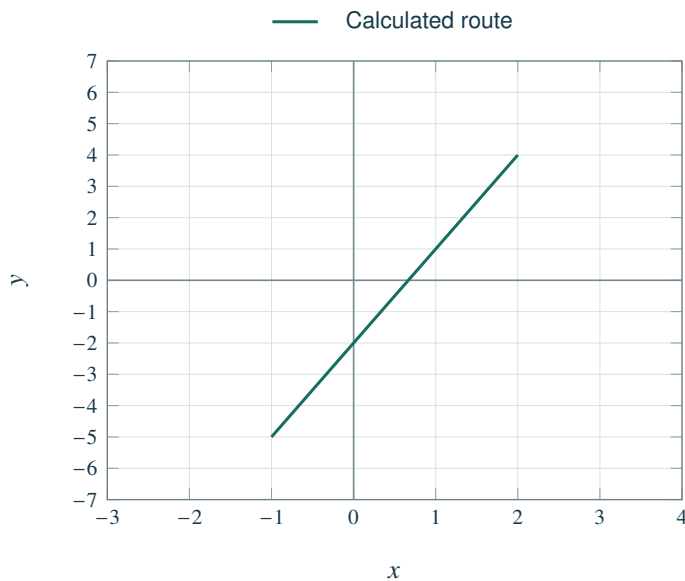
- Coordinates are ordered (x, y) : horizontal position first, vertical position second.
- Substitute each x -value into the equation to obtain its matching y -value.
- Two distinct points determine a straight line; extra points provide a useful check.
- A point lies on a line only when its coordinates satisfy the equation.

Pages 1–8: student resource. Pages 9–10: worked answers.

Worked examples / connect the calculation to the graph

1. Complete and plot a table

For $y = 3x - 2$, find the points when $x = -1, 0$ and 2 .



1. Substitute each x -value: $y = -3 - 2$, $y = 0 - 2$, and $y = 6 - 2$.
2. Pair each result with the x -value that produced it.

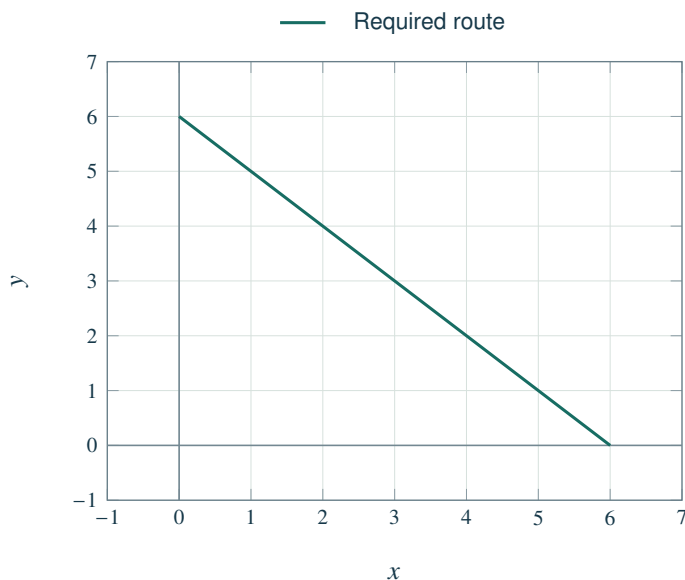
Result

$(-1, -5), (0, -2), (2, 4)$.

Worked example: the table supplies three points on the same straight line.

2. Audit a transmitted point

A beacon reports $(4, 1)$ on $y = -x + 6$. Is it correct?



1. Substitute $x = 4$ to obtain $y = -4 + 6 = 2$.
2. The reported y -value is 1, not 2.

Result

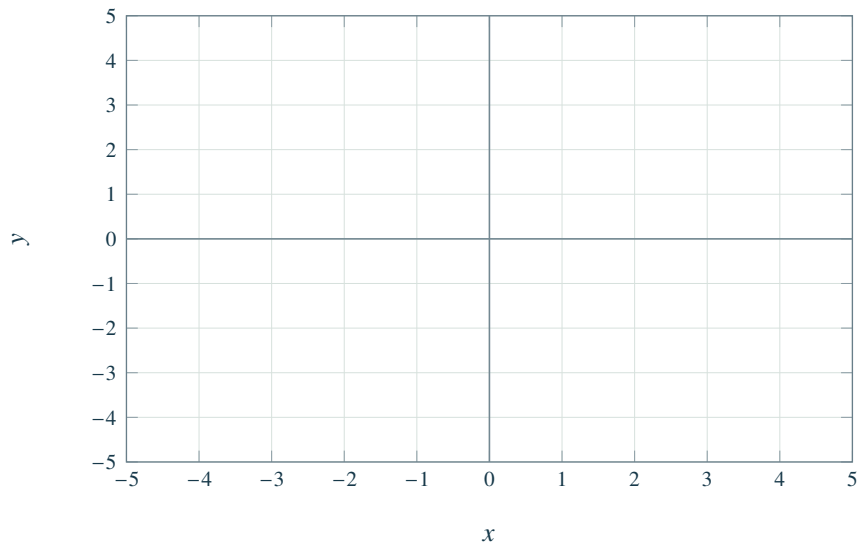
No. At $x = 4$ the correct point is $(4, 2)$.

Worked example: compare a reported beacon with the route at its horizontal position.

Coordinates and tables

Use the graph to record your thinking. Show calculations and include units where needed.

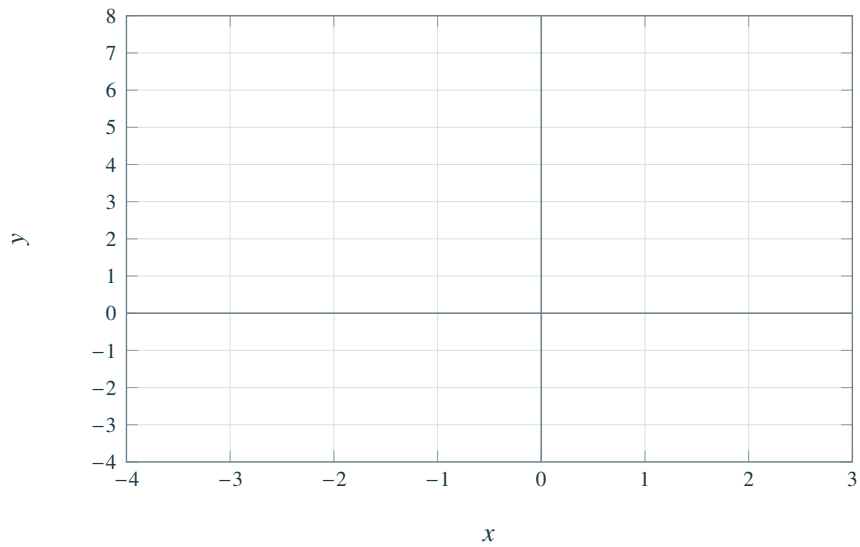
1. Plot $P(-4, 2)$, $Q(3, -3)$, $R(-2, -4)$ and $S(4, 3)$ on the grid. State the quadrant of each point.



Use these labelled axes to construct and check the requested graph.

WORKING / DECISION

2. Complete the y -values for $y = 2x + 3$ when $x = -3, -1, 0$ and 2 . Write the resulting coordinate pairs.



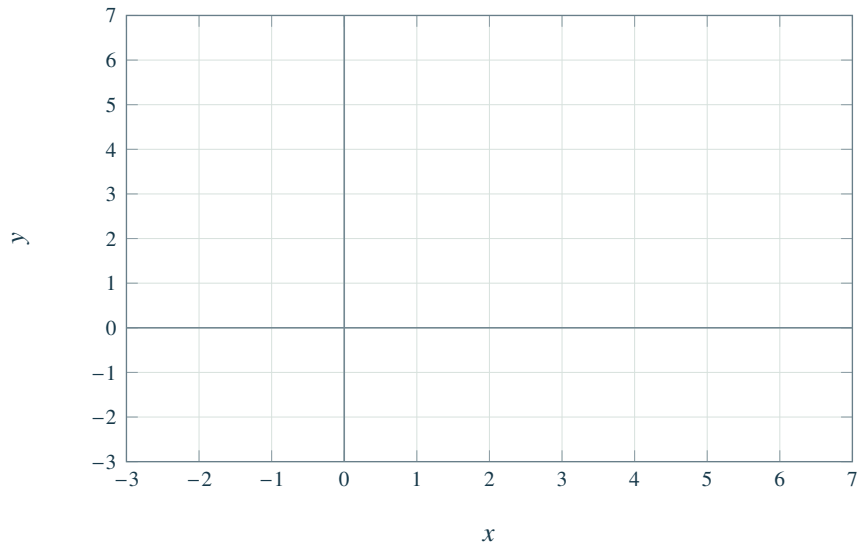
Plot the coordinates from your completed table.

WORKING / DECISION

Coordinates and tables

Use the graph to record your thinking. Show calculations and include units where needed.

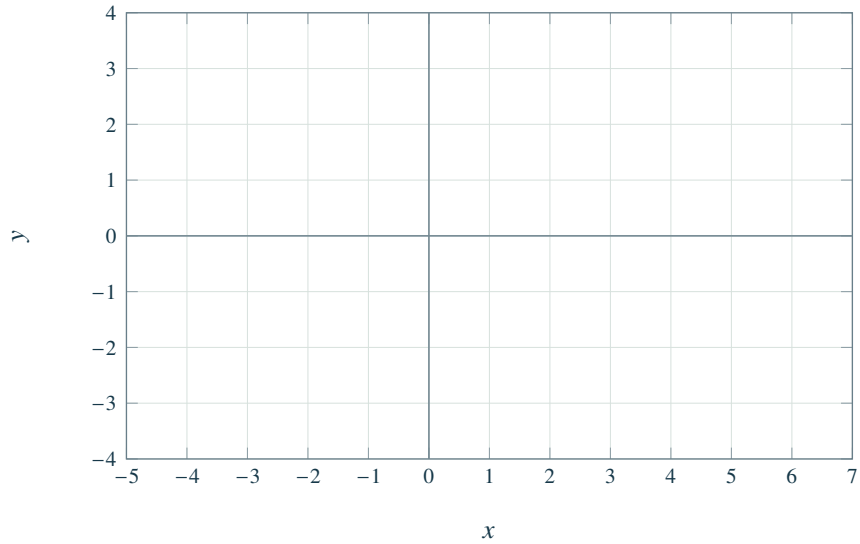
3. Complete the table for $y = -x + 4$ at $x = -2, 0, 3$ and 6 . Which of these points lies below the x -axis?



Place your calculated points and mark any point below the horizontal axis.

WORKING / DECISION

4. For $y = \frac{x}{2} - 1$, calculate y when $x = -4, 0, 2$ and 6 . Explain why these even x -values are convenient.



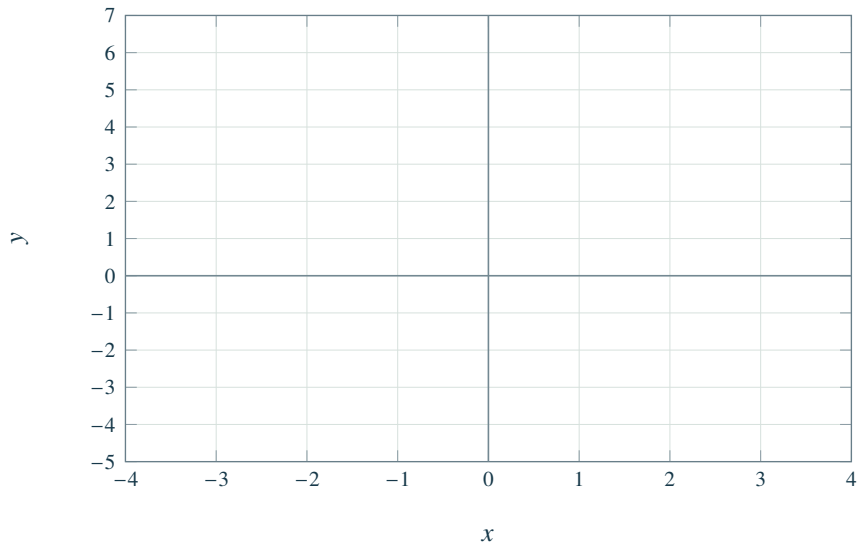
Use the chosen even inputs to build your fractional-rule graph.

WORKING / DECISION

Construct and repair

Use the graph to record your thinking. Show calculations and include units where needed.

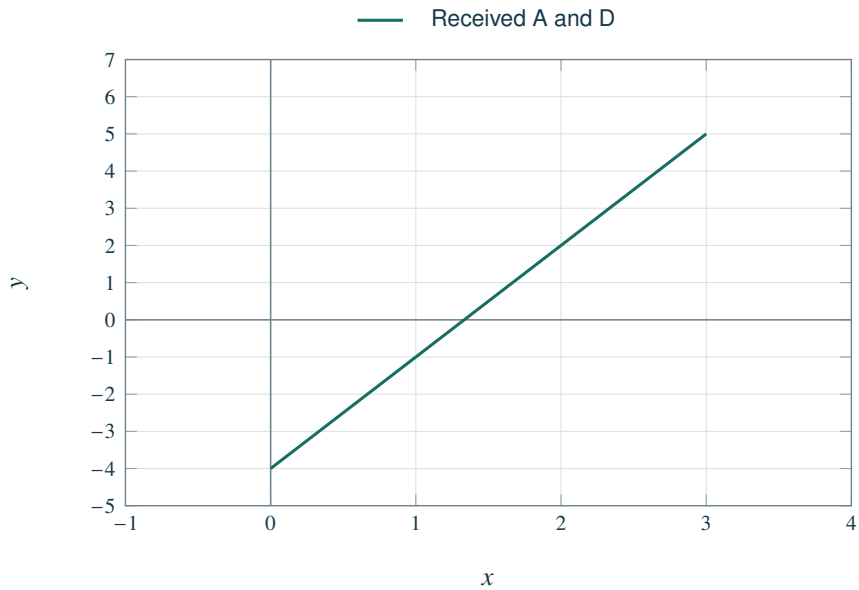
5. Complete a table at $x = -2, 0$ and 2 for $y = -2x + 1$. Plot the three points and draw the straight line.



Use these labelled axes to construct and check the requested graph.

WORKING / DECISION

6. A route should follow $y = 3x - 4$. Received coordinates are $A(0, -4)$, $B(1, -1)$, $C(2, 3)$ and $D(3, 5)$. Identify and correct the faulty coordinate.



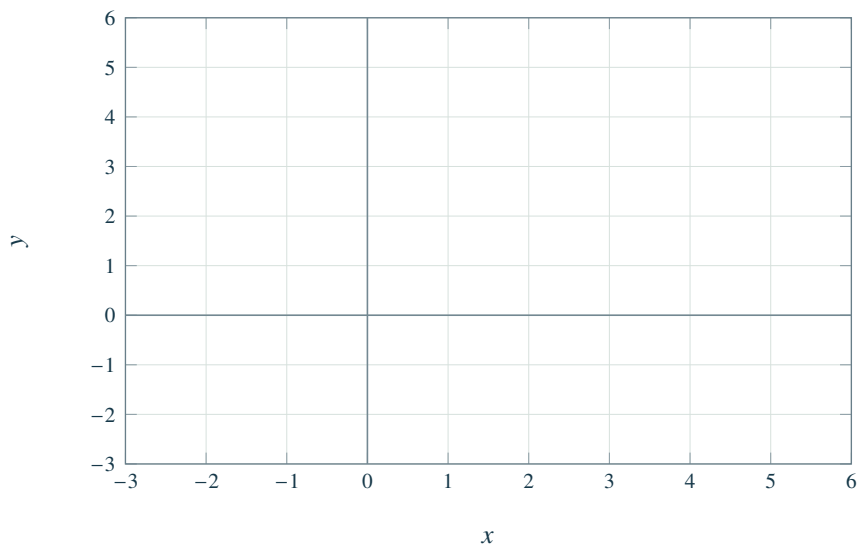
The end records are shown. Check the other received coordinates against the stated rule.

WORKING / DECISION

Construct and repair

Use the graph to record your thinking. Show calculations and include units where needed.

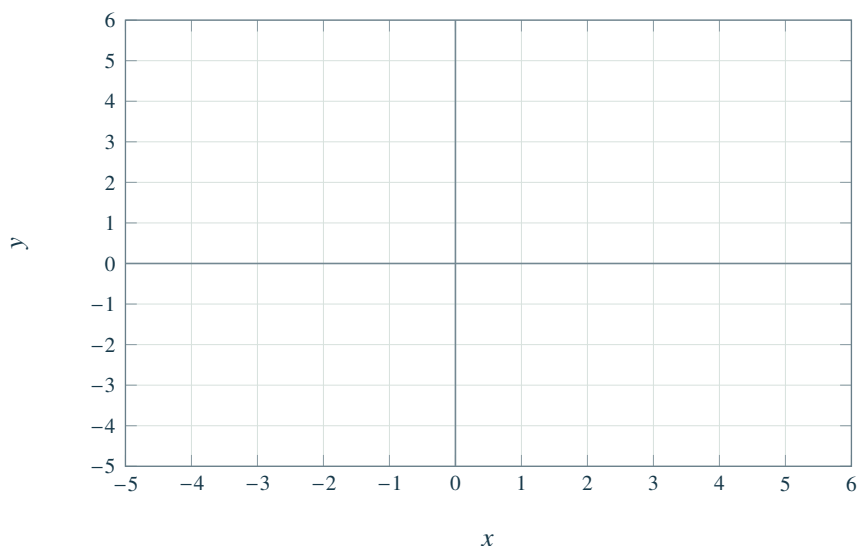
7. Plot the line $x + y = 3$ using $x = -2, 0$ and 5 . Write the equation in a form that makes calculating y easy.



WORKING / DECISION

Use these labelled axes to construct and check the requested graph.

8. A student turns the point $(-3, 5)$ into $(5, -3)$ when plotting. Explain the error and describe how to locate the correct point.



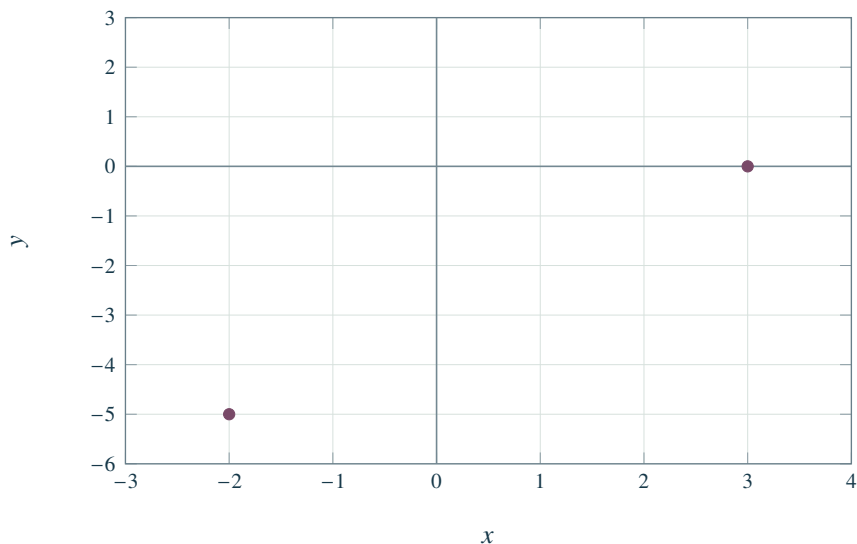
WORKING / DECISION

Plot and label the reported point and the mistakenly reversed point.

Use and test the graph

Use the graph to record your thinking. Show calculations and include units where needed.

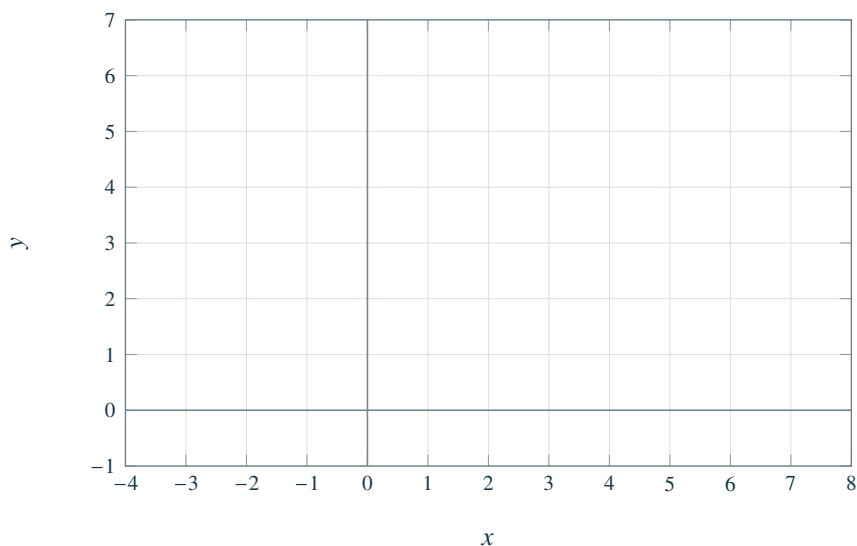
9. Two beacons are $Q(-2, -5)$ and $R(3, 0)$. Which equation fits both: $Ay = 2x - 1$, $By = x - 3$, or $Cy = -x + 3$?



WORKING / DECISION

The two surviving records are supplied. Test the candidate equations without moving these points.

10. The route $y = \frac{x}{2} + 2$ must reach a gate at height $y = 5$. Find the gate's x -coordinate and check it by substitution.



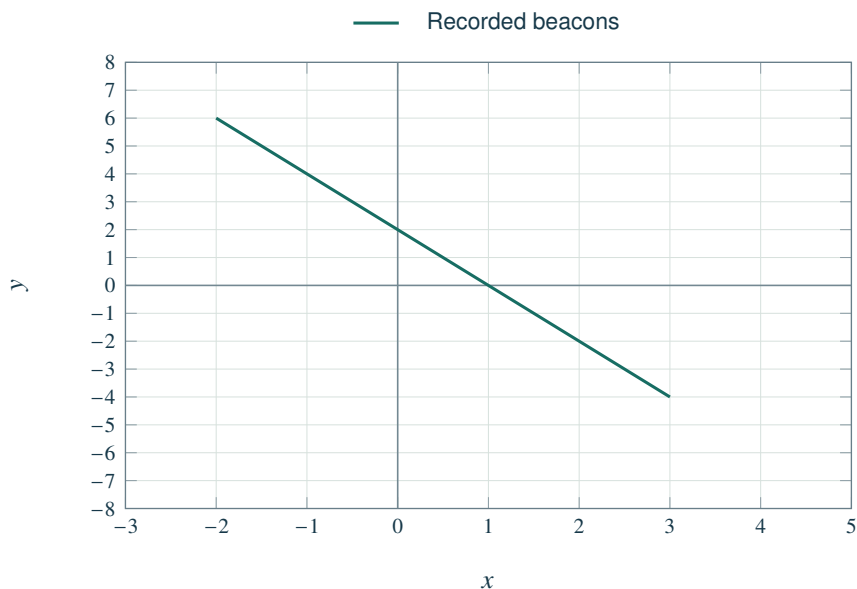
WORKING / DECISION

Draw enough of the route to locate a gate at the stated height.

Use and test the graph

Use the graph to record your thinking. Show calculations and include units where needed.

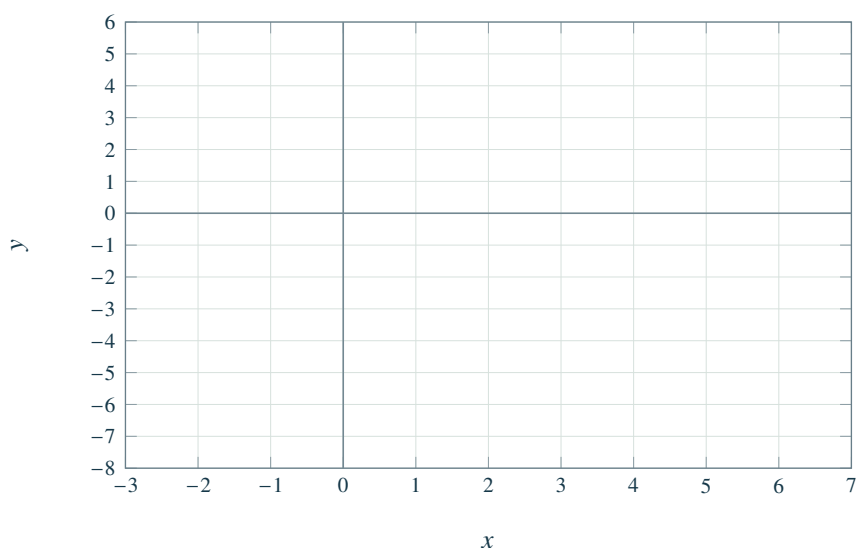
- 11.** A straight line passes through $(-2, 6)$, $(0, 2)$ and $(3, -4)$. Does the beacon $(4, -5)$ continue the same pattern? If not, give its corrected y -value.



WORKING / DECISION

The three reliable records are shown; add and check the proposed extra beacon.

- 12.** For $y = -3\frac{x}{2} + 2$, calculate and plot points at $x = -2, 0$ and 4 . Then find x when $y = -7$.



WORKING / DECISION

Use these labelled axes to construct and check the requested graph.

6C / Worked answers

Teacher appendix. Equivalent exact forms are accepted. Use the reasoning to check student work.

1. P: II; Q: IV; R: III; S: I.

- P has negative x and positive y ; Q has positive x and negative y .
 - R has two negative coordinates; S has two positive coordinates.
-

2. $(-3, -3), (-1, 1), (0, 3), (2, 7)$.

- Substitute each x : $-6 + 3, -2 + 3, 0 + 3, 4 + 3$.
 - The y -values are $-3, 1, 3$ and 7 .
-

3. $(-2, 6), (0, 4), (3, 1), (6, -2)$; only $(6, -2)$ is below.

- Calculate $y = 6, 4, 1$ and -2 respectively.
 - A point is below the x -axis when its y -coordinate is negative.
-

4. $y = -3, -1, 0, 2$; even x -values give integer y -values.

- Divide each x -value by 2, then subtract 1.
 - The coordinates are $(-4, -3), (0, -1), (2, 0), (6, 2)$.
-

5. Points $(-2, 5), (0, 1), (2, -3)$, joined by a straight line.

- Substitution gives $y = 5, 1$ and -3 .
 - The three points align and the line falls from left to right.
-

6. C should be $(2, 2)$.

- At $x = 2$ the equation gives $y = 6 - 4 = 2$.
 - The other three pairs satisfy the equation.
-

6C / Worked answers

Teacher appendix. Equivalent exact forms are accepted. Use the reasoning to check student work.

7. $y = 3 - x$; points $(-2, 5)$, $(0, 3)$, $(5, -2)$.

- Rearrange $x + y = 3$ by subtracting x .
 - Substitute the three x -values and join their plotted points.
-

8. They reversed x and y . Move 3 units left and 5 units up from the origin.

- The first coordinate controls horizontal movement.
 - The second coordinate controls vertical movement.
-

9. B: $y = x - 3$.

- At $x = -2$, B gives -5 ; at $x = 3$, B gives 0 .
 - A fits Q but not R; C does not fit either pair.
-

10. $x = 6$; gate $(6, 5)$.

- $5 = \frac{x}{2} + 2$ gives $\frac{x}{2} = 3$ and $x = 6$.
 - Substitution gives $\frac{6}{2} + 2 = 5$.
-

11. No; at $x = 4$ the correct y -value is -6 .

- The pattern decreases y by 2 for every increase of 1 in x .
 - From $(3, -4)$, moving one unit right gives $(4, -6)$.
-

12. $(-2, 5)$, $(0, 2)$, $(4, -4)$; $x = 6$ when $y = -7$.

- Substitution gives the three coordinates.
 - Solve $-7 = -3\frac{x}{2} + 2$: $-9 = -3\frac{x}{2}$, so $x = 6$.
-

CHAPTER 6 / 6D

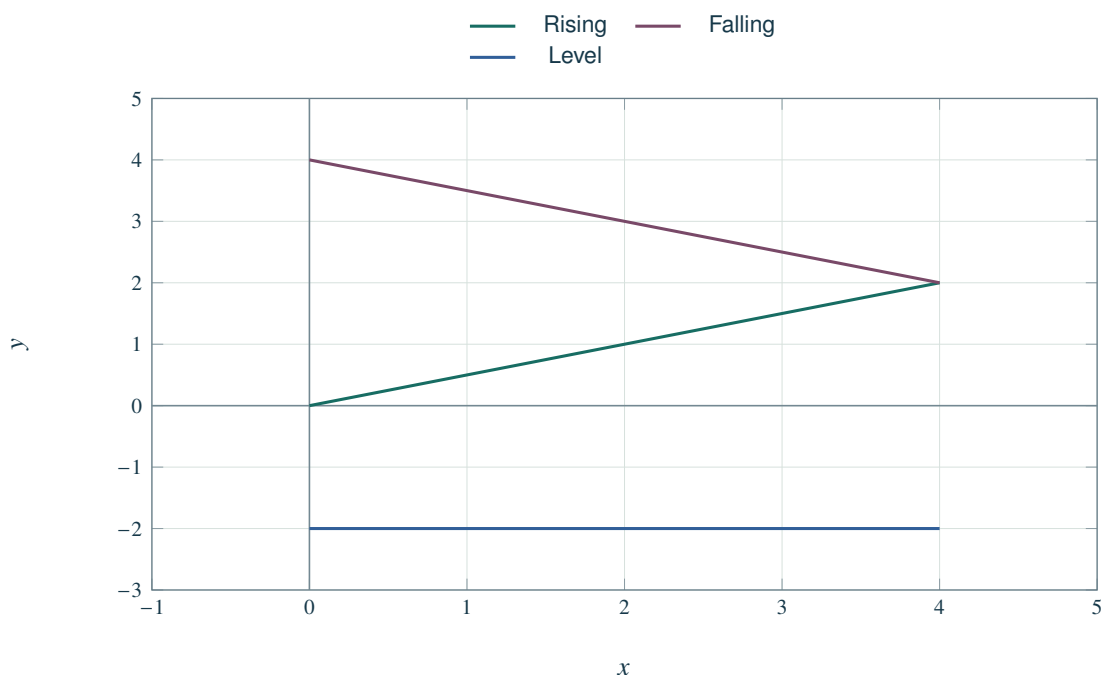
Exploring gradient

Name: _____ Class: _____ Date: _____

Measure steepness with signed rise and run. Compare slopes on equal scales, distinguish horizontal and vertical lines, and use gradient in practical design decisions.

What you will learn

- Calculate gradient from graphs or two points.
- Interpret positive, negative, zero and undefined gradients.
- Express gradients as fractions, decimals and percentages in context.

See the mathematics

Learning map: compare the signed height change for the same horizontal run.

Key ideas

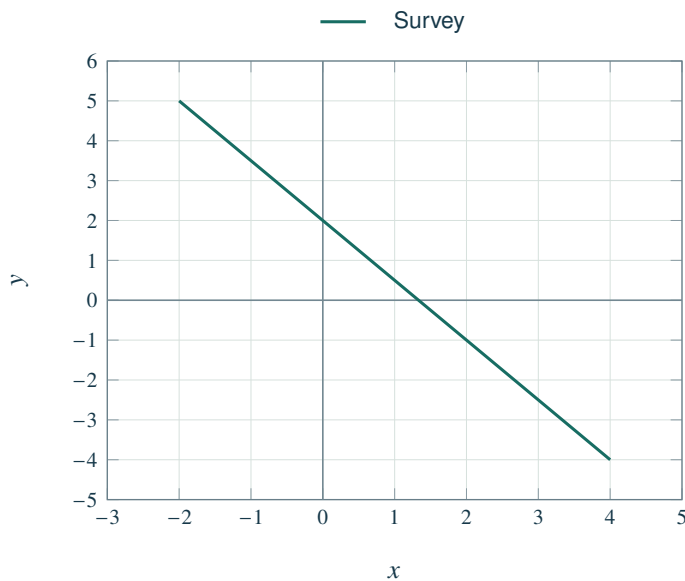
- Gradient is $m = \frac{y_2 - y_1}{x_2 - x_1}$. Keep the same point order in both differences.
- A positive gradient rises from left to right; a negative gradient falls.
- A horizontal line has $m = 0$; a vertical line has undefined gradient.
- Percentage gradient is $100m\%$. Compare $|m|$ when judging steepness.

Pages 1–8: student resource. Pages 9–10: worked answers.

Worked examples / connect the calculation to the graph

1. A signed gradient

Find the gradient through $A(-2, 5)$ and $B(4, -4)$.



1. Signed rise = $-4 - 5 = -9$; run = $4 - (-2) = 6$.

2. $m = \frac{-9}{6} = \frac{-3}{2}$.

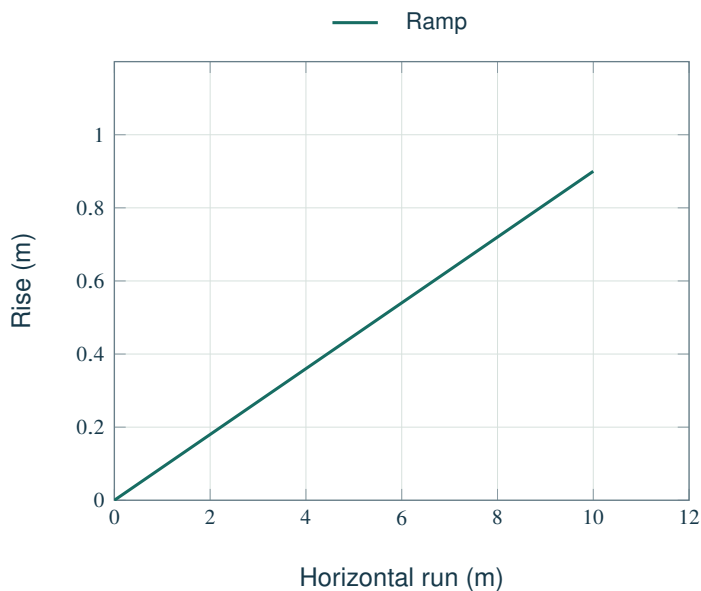
Result

$m = \frac{-3}{2}$. The line falls left to right.

Worked example: preserve the same point order in both coordinate changes.

2. Design a gentle ramp

A ramp rises 0.9 m over a 10 m horizontal run. Express its gradient as a decimal and percentage.



1. Gradient = rise/run = $\frac{0.9}{10} = 0.09$.

2. Multiply by 100 to convert to percentage gradient.

Result

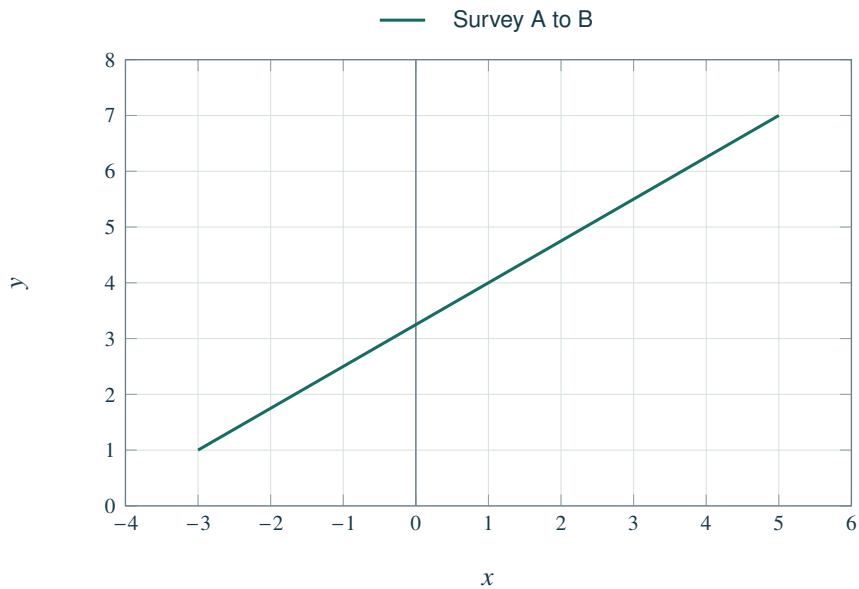
Gradient $0.09 = 9\%$.

Worked example: express one rise-to-run ratio as a decimal and percentage.

Signed rise and run

Use the graph to record your thinking. Show calculations and include units where needed.

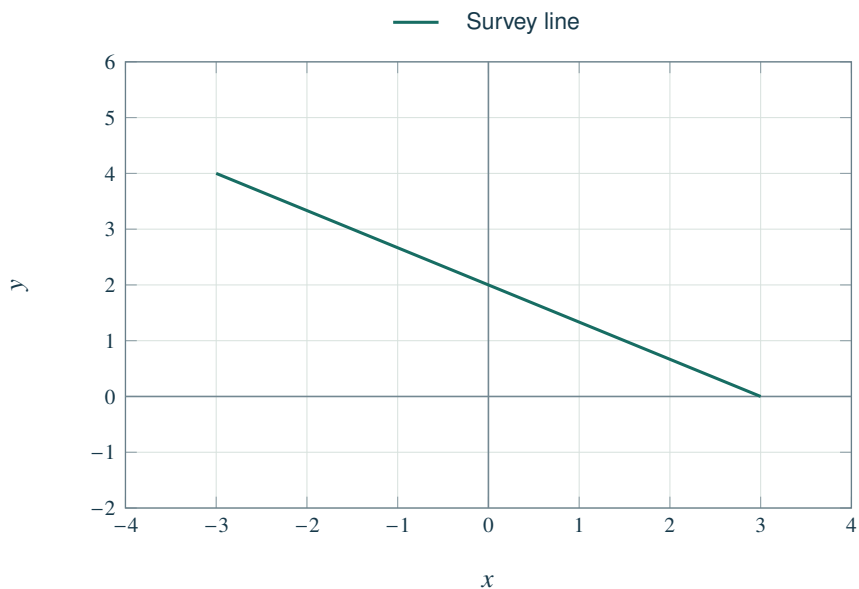
1. Find the signed rise, run and gradient from $A(-3, 1)$ to $B(5, 7)$.



Use the same two points for both signed coordinate changes.

WORKING / DECISION

2. Find the gradient of the line shown. State whether it rises or falls from left to right.



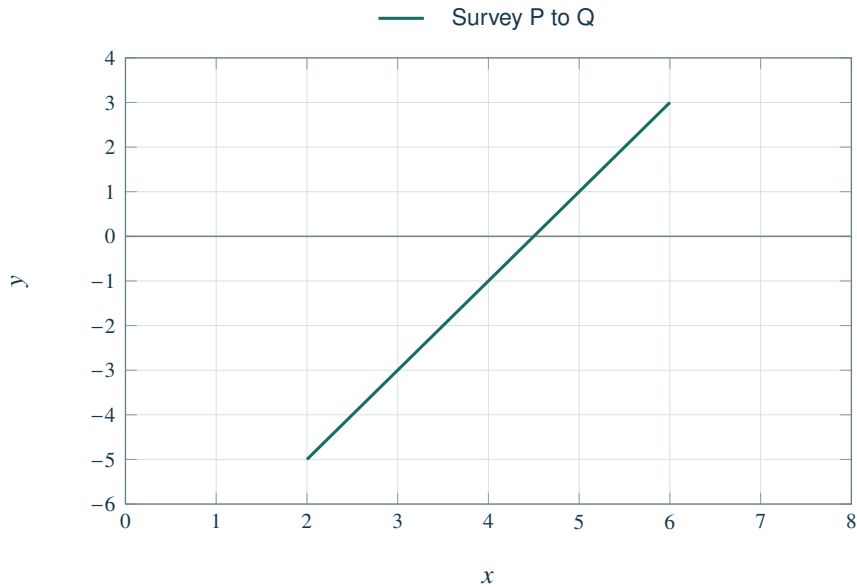
Read the quantities, scales and labelled line before answering.

WORKING / DECISION

Signed rise and run

Use the graph to record your thinking. Show calculations and include units where needed.

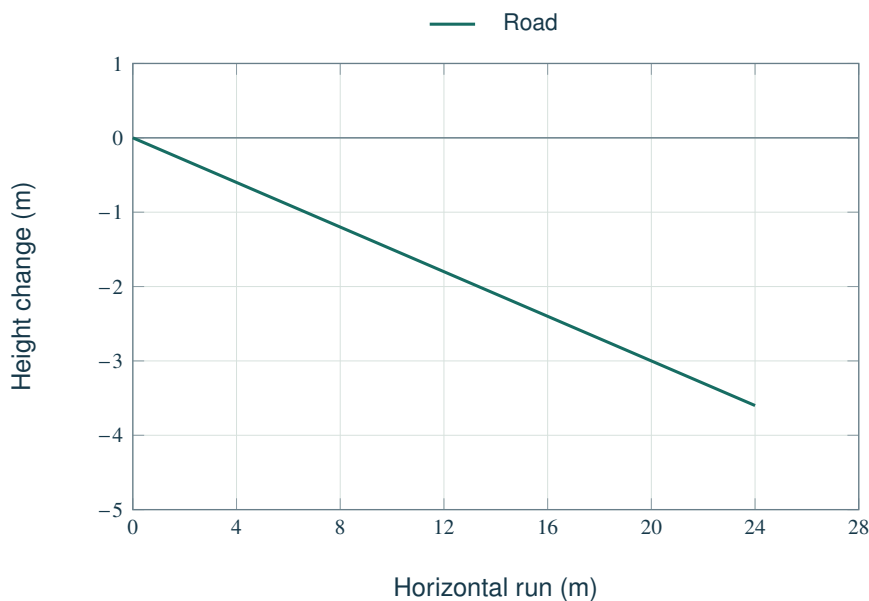
3. Two survey points are $P(2, -5)$ and $Q(6, 3)$. Calculate the gradient using P first, then Q first. Explain why the result is unchanged.



WORKING / DECISION

Calculate in each direction along the same line.

4. A road falls 3.6 m while moving 24 m horizontally to the right. Give its signed gradient as a fraction and decimal.



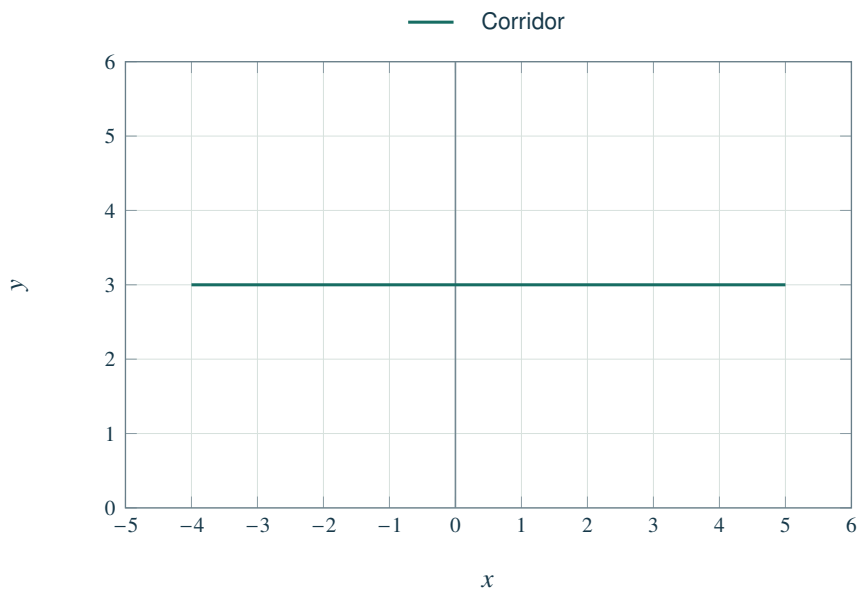
WORKING / DECISION

The vertical change is measured relative to the starting road height.

Special lines and comparisons

Use the graph to record your thinking. Show calculations and include units where needed.

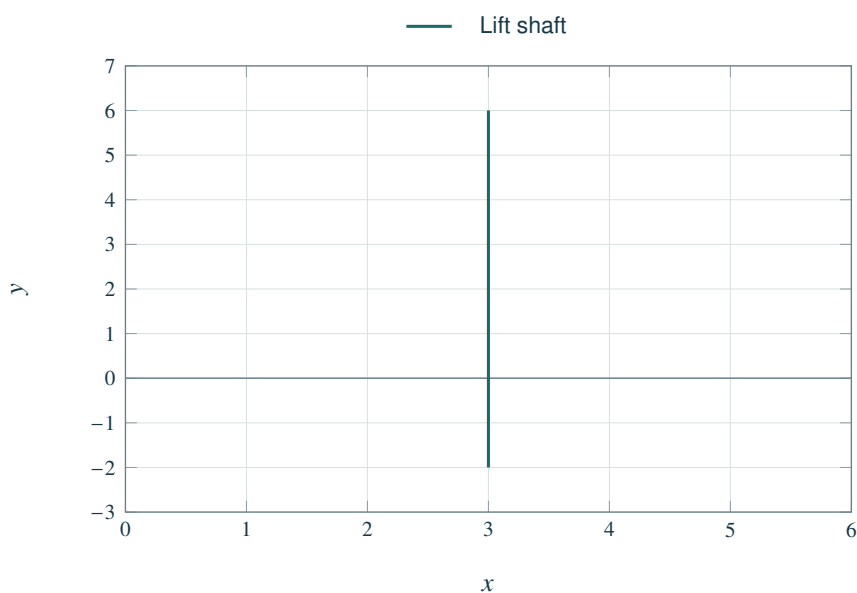
5. Find the gradient through $(-4, 3)$ and $(5, 3)$. Explain the result using rise and run.



Interpret the horizontal and vertical changes separately.

WORKING / DECISION

6. A lift shaft joins $(3, -2)$ and $(3, 6)$. State its gradient and explain why it is not zero.



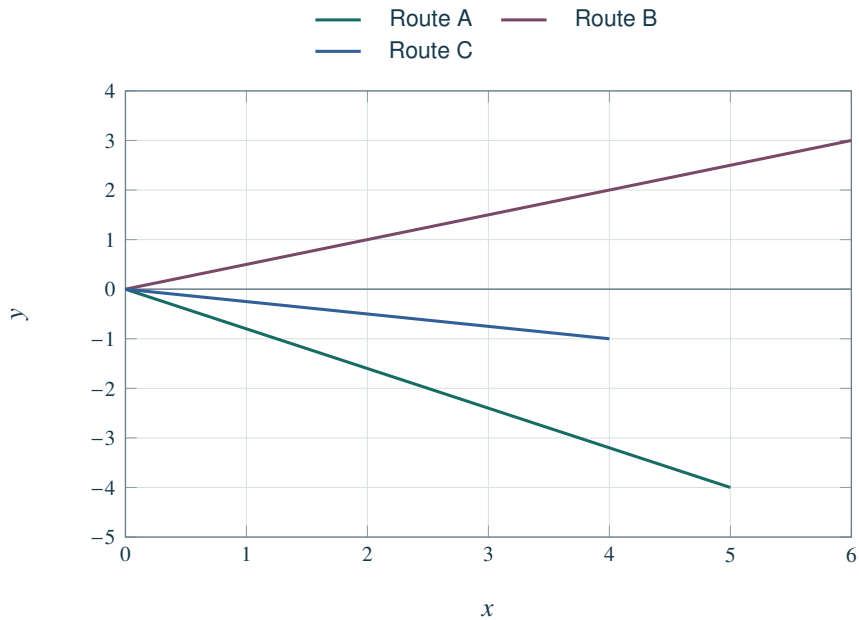
The shaft endpoints share one coordinate. Use that information to describe its gradient.

WORKING / DECISION

Special lines and comparisons

Use the graph to record your thinking. Show calculations and include units where needed.

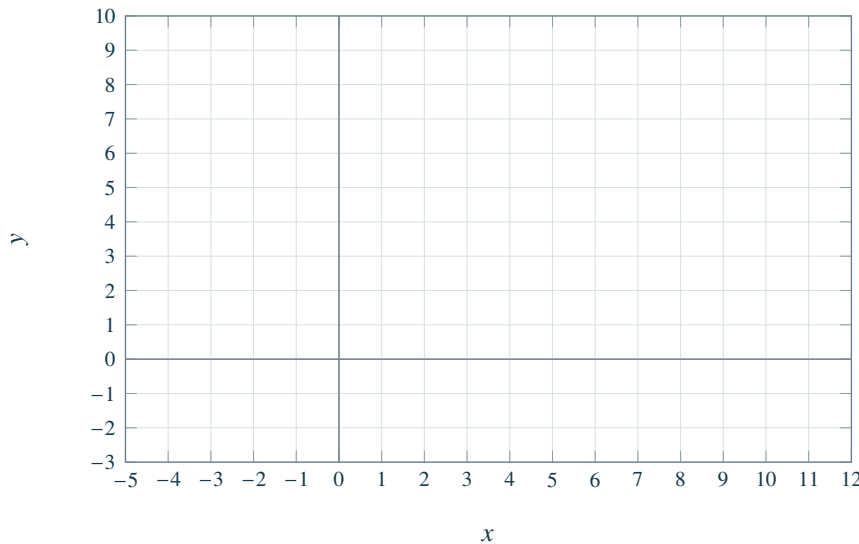
7. Routes have gradients $A = \frac{-4}{5}$, $B = \frac{1}{2}$ and $C = \frac{-1}{4}$. Order them from gentlest to steepest.



WORKING / DECISION

All three routes use the same scales so their steepness can be compared.

8. A line has gradient $\frac{3}{5}$ and passes through $(1, 2)$. Find the point on the line 10 units to the right. Find the point 5 units to the left.



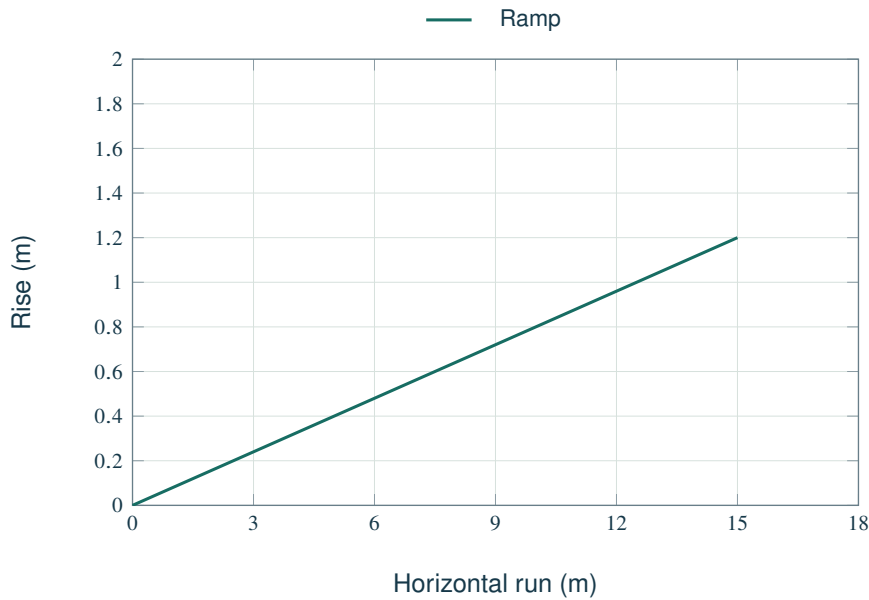
WORKING / DECISION

Start at the given point and construct both requested moves.

Engineering decisions

Use the graph to record your thinking. Show calculations and include units where needed.

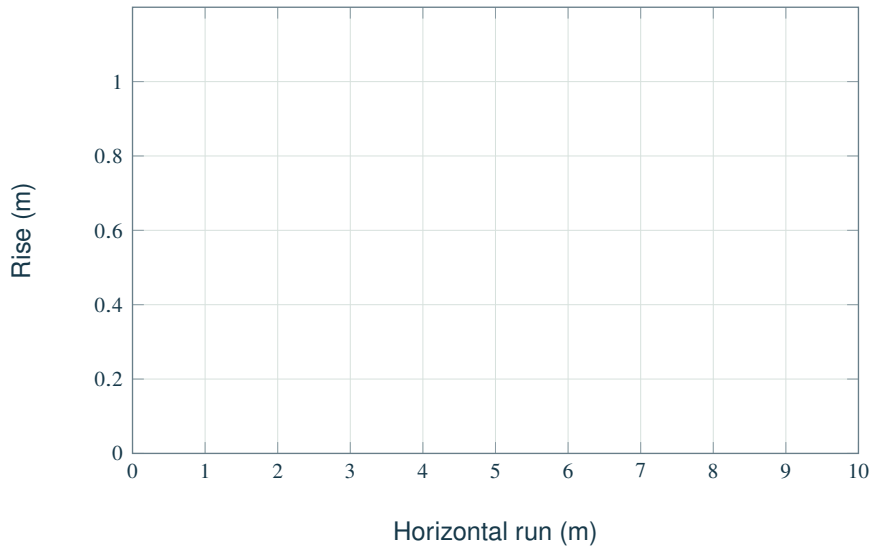
9. A ramp rises 1.2 m over a 15 m run. Express its gradient as a fraction, decimal and percentage.



WORKING / DECISION

Read the rise and run of the ramp; express the same slope in three forms.

10. For this puzzle, a service ramp may have gradient no greater than $\frac{1}{10}$. It rises 0.8 m. Find its minimum horizontal run. Does it fit within 9 m?



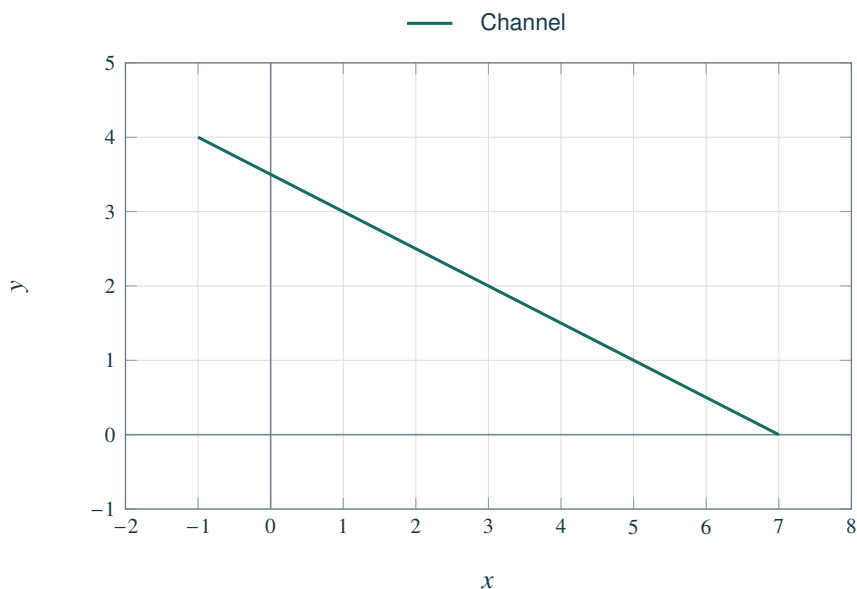
WORKING / DECISION

Draw your proposed ramp to the 0.8 m platform and check it against the stated design limit.

Engineering decisions

Use the graph to record your thinking. Show calculations and include units where needed.

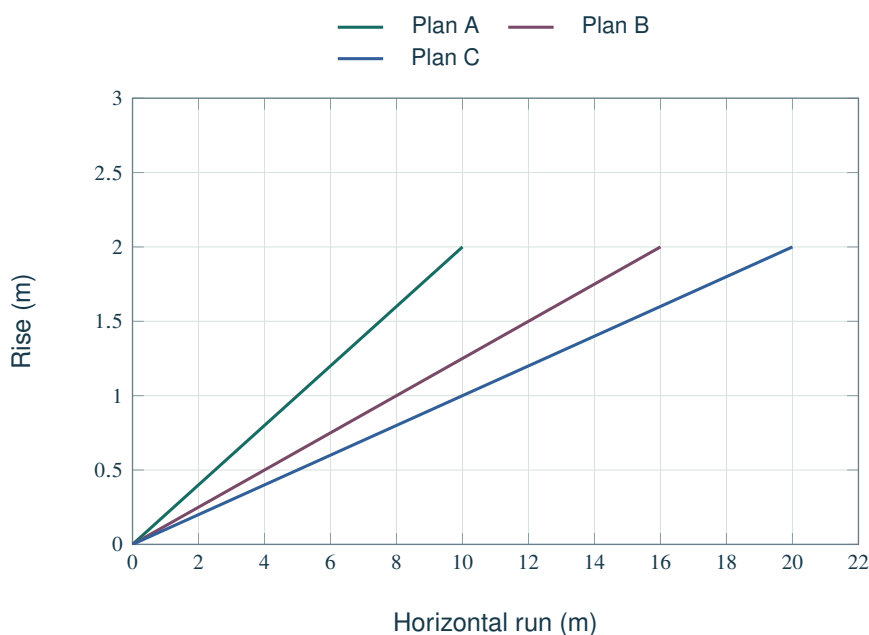
- 11.** A channel runs from $(-1, 4)$ to $(7, 0)$. A surveyor reports gradient $+\frac{1}{2}$. Give the correct gradient and explain the likely error.



WORKING / DECISION

Compare the surveyor's stated sign with the actual direction from left to right.

- 12.** A bridge rises 2 m. Plans have runs A: 10m, B: 16m, C: 20 m. The gradient must be at most $\frac{1}{8}$. Each metre of run costs \$12 and the budget is \$210. Choose a plan and give its gradient, percentage and cost.



WORKING / DECISION

Compare the profiles, then use the price per metre and budget from the question.

6D / Worked answers

Teacher appendix. Equivalent exact forms are accepted. Use the reasoning to check student work.

1. Rise 6; run 8; gradient $\frac{3}{4}$.

- Rise = $7 - 1 = 6$; run = $5 - (-3) = 8$.
 - $m = \frac{6}{8} = \frac{3}{4}$.
-

2. $m = \frac{-2}{3}$; it falls.

- Using $(-3, 4)$ and $(3, 0)$, rise = $0 - 4 = -4$ and run = 6.
 - $m = \frac{-4}{6} = \frac{-2}{3}$.
-

3. Both calculations give $m = 2$.

- P to Q: $(3 - (-5))/(6 - 2) = \frac{8}{4} = 2$.
 - Q to P: $\frac{-5-3}{2-6} = \frac{-8}{-4} = 2$; both signs reverse.
-

4. $\frac{-3}{20} = -0.15$.

- Rise is $-3.6m$ because the road falls.
 - $m = \frac{-3.6}{24} = -0.15 = \frac{-3}{20}$.
-

5. 0; it is a horizontal line.

- Rise = $3 - 3 = 0$; run = $5 - (-4) = 9$.
 - $m = \frac{0}{9} = 0$.
-

6. Undefined; the run is zero.

- Rise = 8, while run = $3 - 3 = 0$.
 - The quotient $\frac{8}{0}$ is undefined; zero gradient would require zero rise instead.
-

6D / Worked answers

Teacher appendix. Equivalent exact forms are accepted. Use the reasoning to check student work.

7. C, B, A.

- Compare absolute values: $\frac{1}{4} = 0.25$, $\frac{1}{2} = 0.5$, $\frac{4}{5} = 0.8$.
 - The negative signs describe falling direction, not reduced steepness.
-

8. (11, 8); (−4, −1).

- For a run of 10, rise = $(\frac{3}{5}) \times 10 = 6$.
 - For a run of −5, rise = $(\frac{3}{5}) \times (-5) = -3$.
-

9. $\frac{2}{25} = 0.08 = 8\%$.

- $m = \frac{1.2}{15} = 0.08$.
 - $0.08 \times 100 = 8\%$.
-

10. Minimum run 8 m; yes.

- $0.8/\text{run} \leq \frac{1}{10}$ implies run ≥ 8 .
 - An 8 m run fits within the 9 m space.
-

11. $-\frac{1}{2}$; the surveyor ignored the falling direction or mixed point orders.

- Rise = $0 - 4 = -4$ and run = $7 - (-1) = 8$.
 - $m = \frac{-4}{8} = -\frac{1}{2}$.
-

12. Plan B; gradient $\frac{1}{8} = 12.5\%$; cost \$192.

- A has gradient $\frac{2}{10} = \frac{1}{5}$, which is too steep.
 - B meets the limit and costs $16 \times 12 = \$192$; C costs \$240, above budget.
-

CHAPTER 6 / 6E

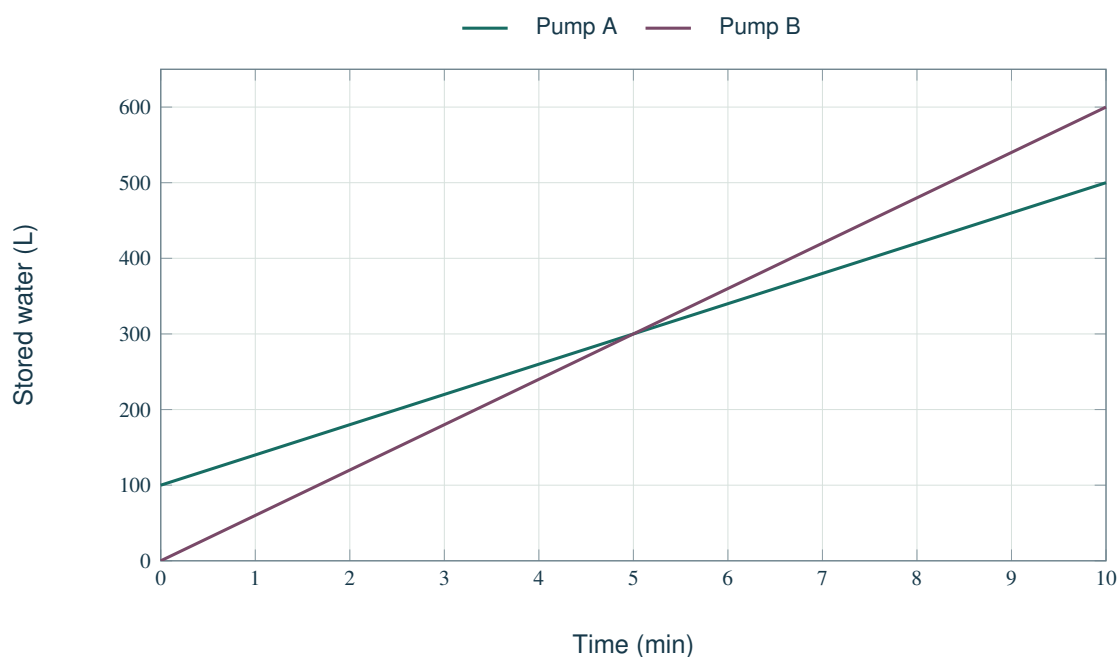
Rates from graphs

Name: _____ Class: _____ Date: _____

Use the gradient of a contextual graph as a rate. Keep the units attached, compare like with like, and distinguish a starting amount from a change.

What you will learn

- Calculate contextual rates from graph intervals.
- Convert units before comparing rates.
- Use positive and negative rates to plan an operation.

See the mathematics

Learning map: a higher starting amount is different from a greater rate of change.

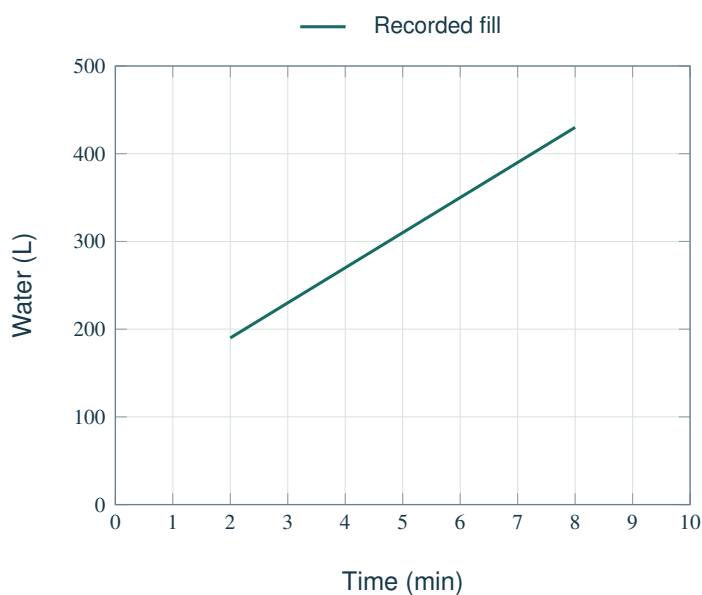
Key ideas

- Rate is $\frac{\text{change in vertical quantity}}{\text{change in horizontal quantity}}$.
- Rate units come from the vertical-axis unit divided by the horizontal-axis unit.
- A starting amount changes the graph's height but not its rate.
- Convert time or quantity units before comparing specifications or forecasting.

Worked examples / connect the calculation to the graph

1. A filling rate

A tank contains 190 L at 2 min and 430 L at 8 min. Find the fill rate.



$$1. \text{ Change in water} = 430 - 190 = 240 \text{ L};$$

$$\text{change in time} = 8 - 2 = 6 \text{ min.}$$

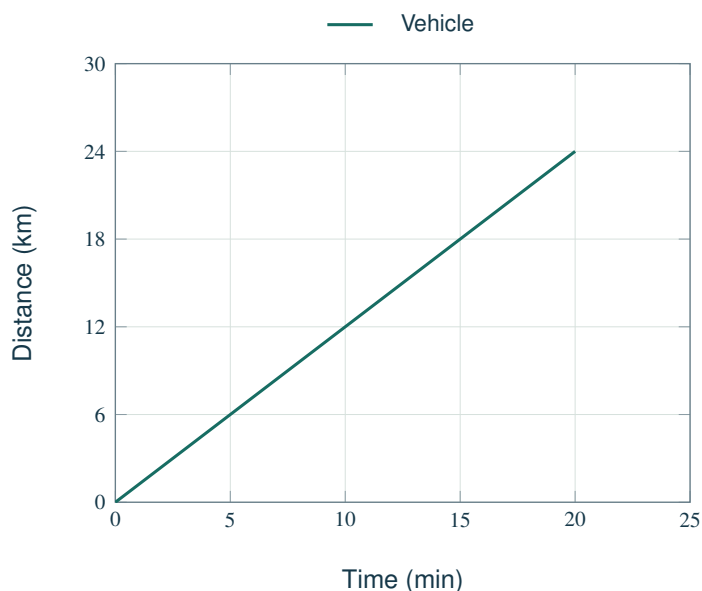
$$2. \text{ Rate} = \frac{240}{6} = 40 \text{ L/min.}$$

Result
40 L/min.

Worked example: the two measurements define the observed rate.

2. Convert a transport rate

A test vehicle travels 24 km in 20 min. Find its speed in km/min and km/h.



$$1. \text{ Speed} = \frac{24}{20} = 1.2 \text{ km/min.}$$

$$2. \text{ Multiply by 60 minutes per hour: } 1.2 \times 60 = 72 \text{ km/h.}$$

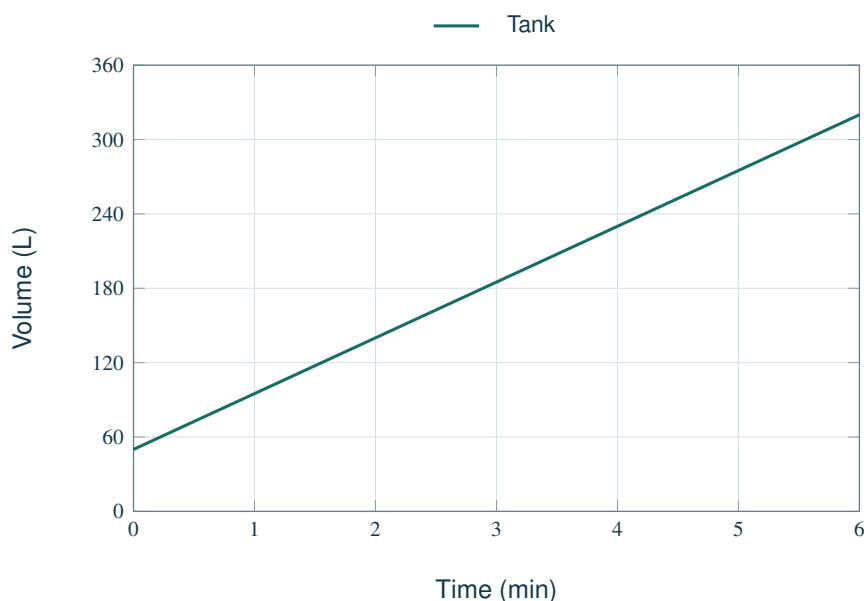
Result
1.2 km/min = 72 km/h.

Worked example: the graph uses minutes; the same journey can also be expressed per hour.

Rates and their units

Use the graph to record your thinking. Show calculations and include units where needed.

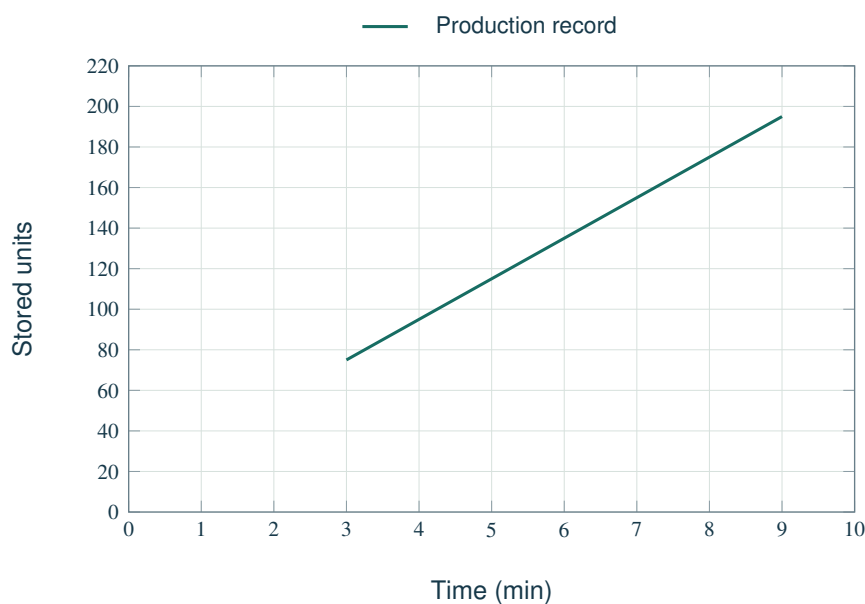
1. The graph shows a tank filling. Find the rate between 2 and 6 minutes, and state the correct units.



WORKING / DECISION

Read the quantities, scales and labelled line before answering.

2. A production graph records 75 units at 3 min and 195 units at 9 min. Find the production rate. Explain why $\frac{195}{9}$ is not the correct calculation.



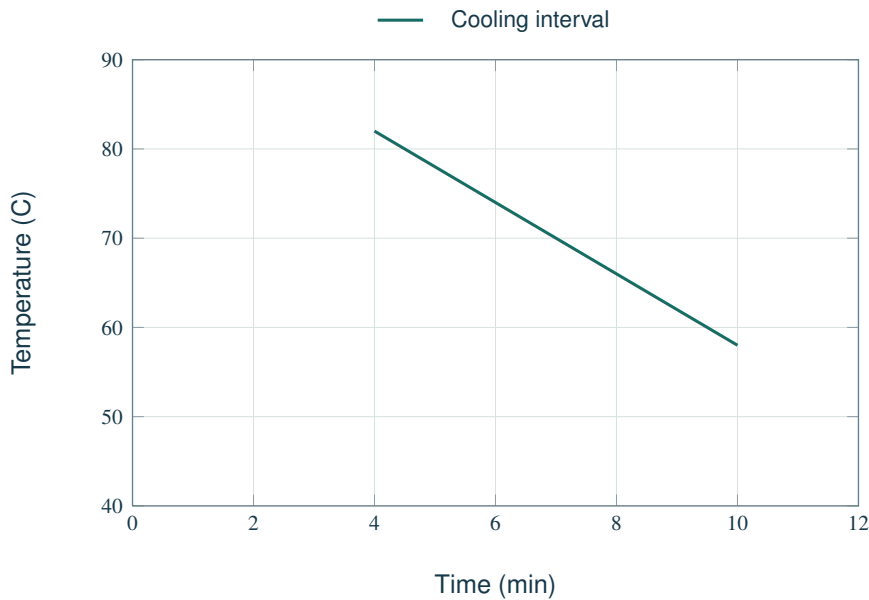
WORKING / DECISION

The rate comes from the recorded interval. The line need not start at the origin.

Rates and their units

Use the graph to record your thinking. Show calculations and include units where needed.

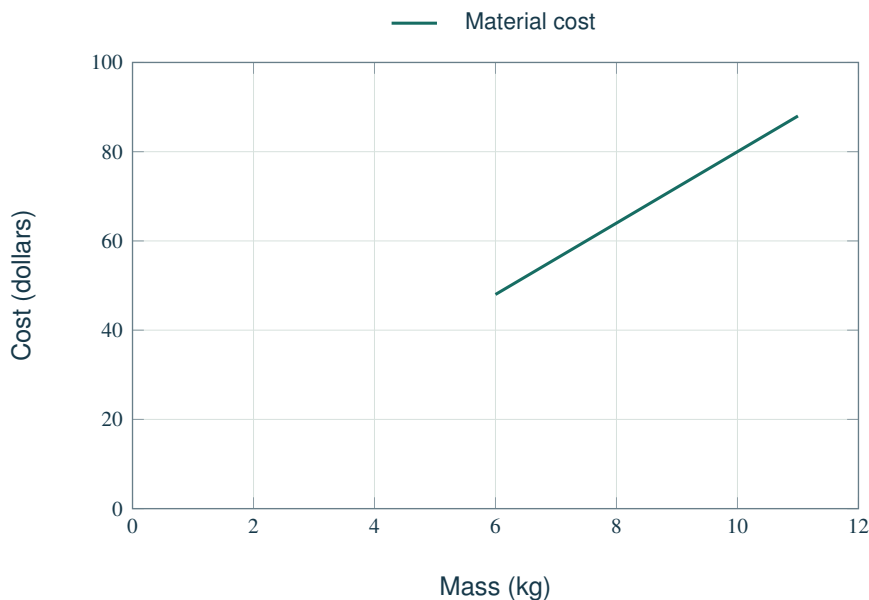
3. A chamber cools from 82°C at 4 min to 58°C at 10 min. Find its signed rate of temperature change.



WORKING / DECISION

Use the temperature and time scales to interpret the signed change.

4. The cost of material rises from \$48 for 6 kg to \$88 for 11 kg. Find the cost per additional kilogram.



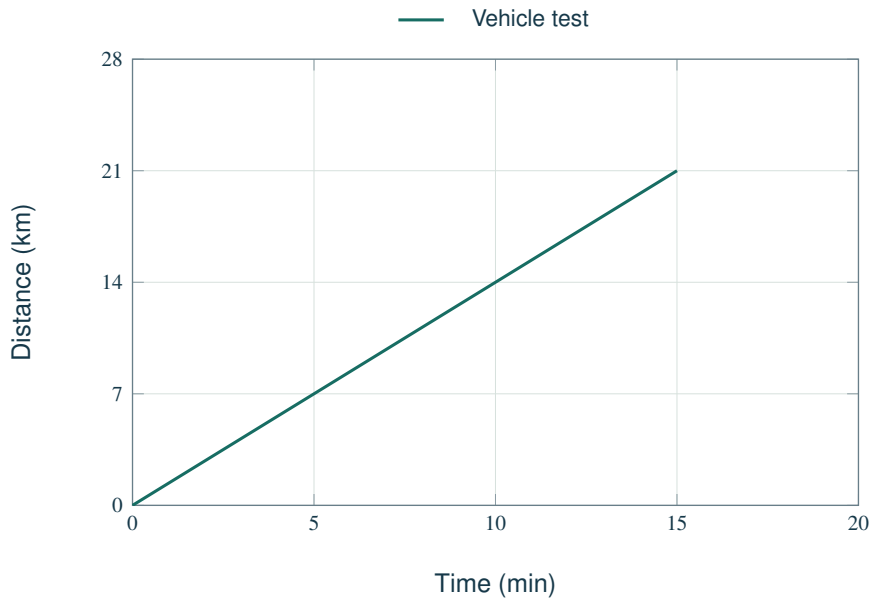
WORKING / DECISION

Compare changes in cost and mass across the same interval.

Convert and compare

Use the graph to record your thinking. Show calculations and include units where needed.

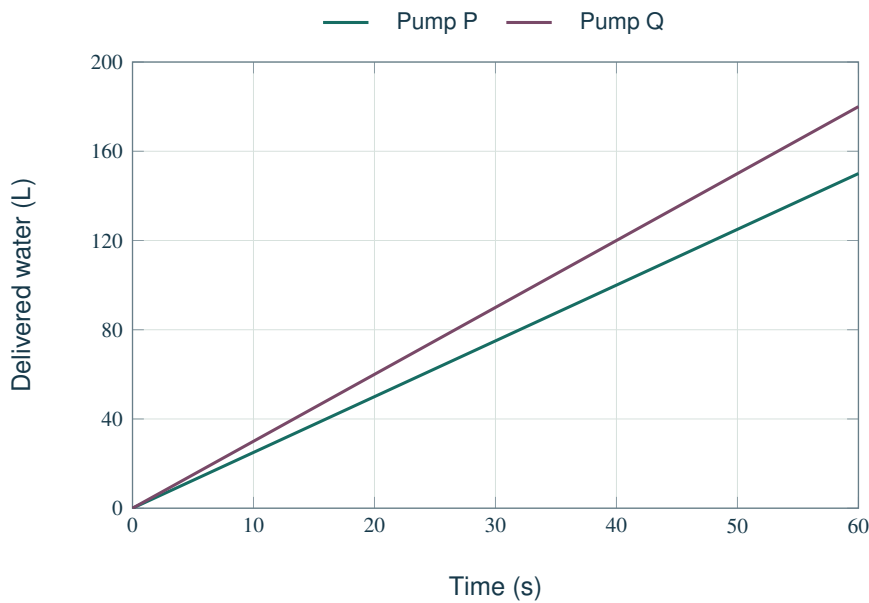
5. A vehicle travels 21 km in 15 min. Find its speed in km/min and km/h.



WORKING / DECISION

The graph uses minutes. Report the same speed in each requested unit.

6. Pump P delivers 2.5 L/s. Pump Q delivers 180 L/min. Which pump has the greater rate and by how much in L/min?



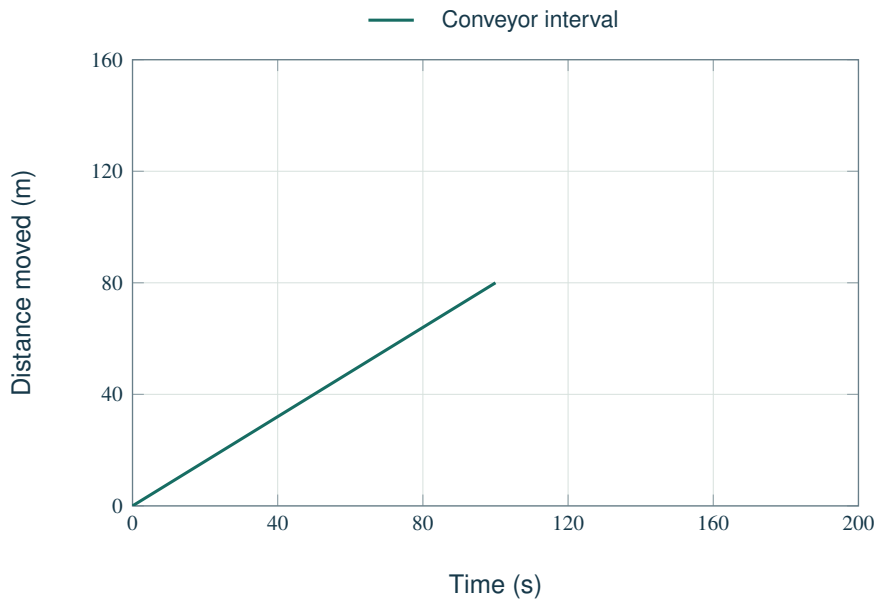
WORKING / DECISION

Both pump outputs are shown on the same time scale for comparison.

Convert and compare

Use the graph to record your thinking. Show calculations and include units where needed.

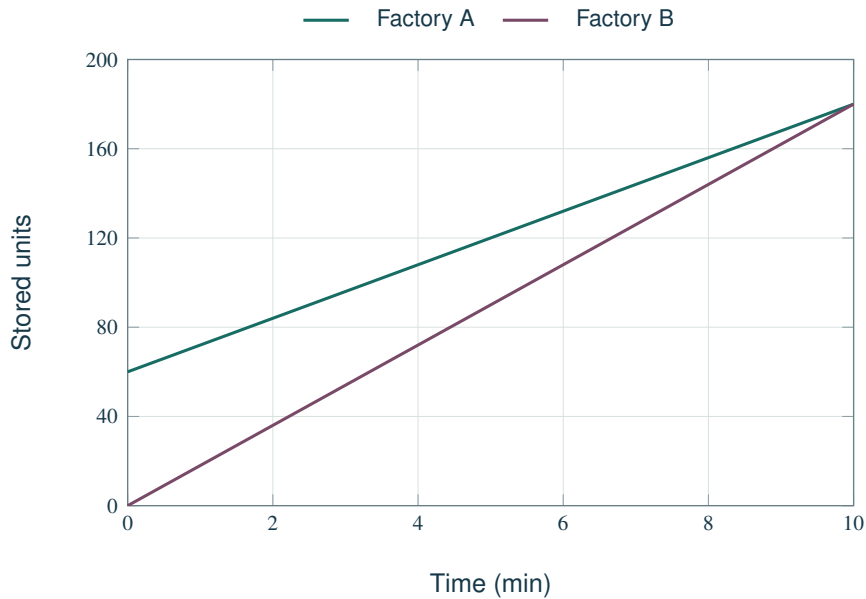
7. A conveyor moves at 0.8 m/s. Express this rate in m/min and find the time required to move an item 144 m.



WORKING / DECISION

Continue the conveyor's recorded movement yourself to plan the requested journey.

8. The graph shows stored output from two factories. Find each production rate and identify the faster producer, even if it begins with less stock.



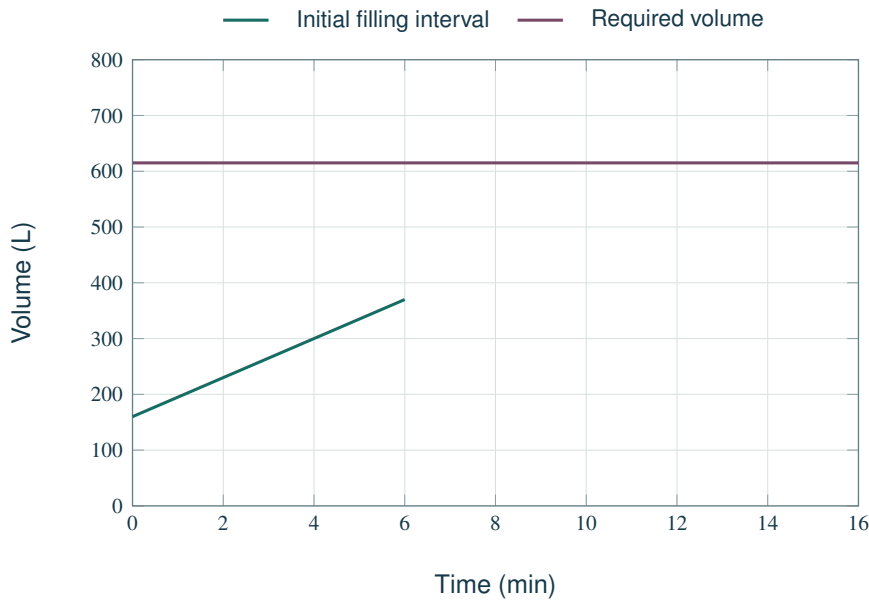
WORKING / DECISION

Read the quantities, scales and labelled line before answering.

Build an operating plan

Use the graph to record your thinking. Show calculations and include units where needed.

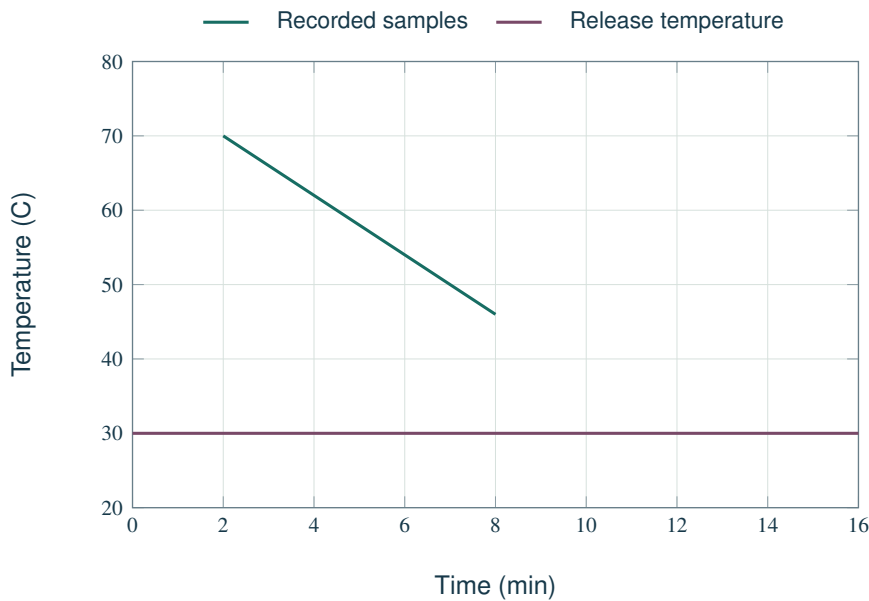
9. A reservoir contains 160 L at time zero and fills at 35 L/min. At what time will it contain 615 L?



WORKING / DECISION

Extend your own filling plan to the required-volume line.

10. A cooling chamber records 70°C at 2 min and 46°C at 8 min. Assuming the same rate continues, when will it reach 30°C from process start?



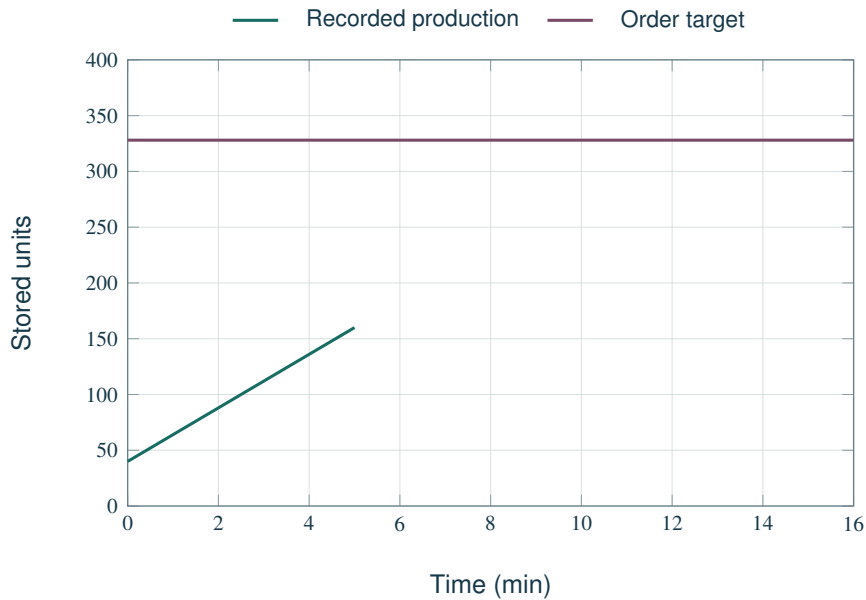
WORKING / DECISION

Only the recorded cooling interval is drawn. Add your proposed release point.

Build an operating plan

Use the graph to record your thinking. Show calculations and include units where needed.

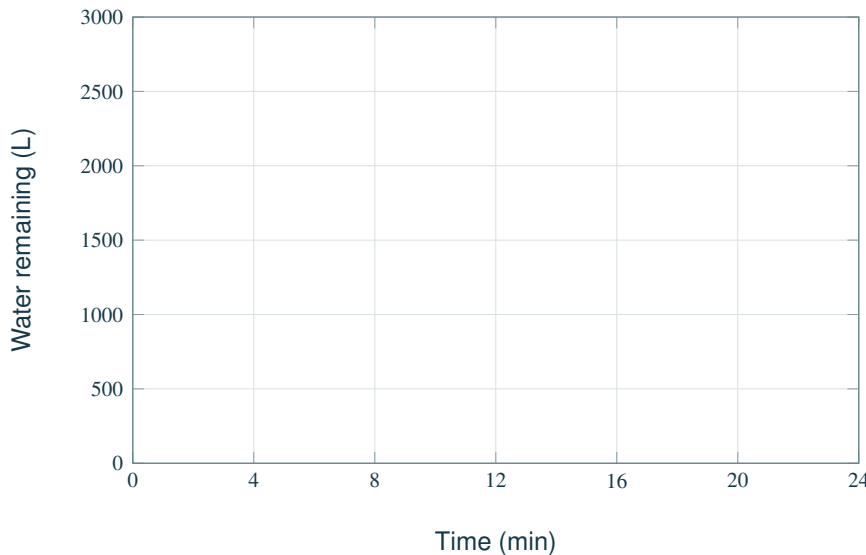
11. A stock graph begins with 40 units and rises at 24 units/min. An order needs 328 units. Find the completion time. How many units were newly produced?



WORKING / DECISION

Distinguish existing stock from newly made units when you extend this line.

12. A basin holds 2700 L and must be empty in 20 min. Budget: \$75. Pump A costs \$40 at 120 L/min. B costs \$65 at 2.5 L/s. C costs \$90 at 12 kL/h. Select a valid pump and give the emptying time.



WORKING / DECISION

Sketch a drainage plan for each candidate after making the units consistent. Mark the deadline.

6E / Worked answers

Teacher appendix. Equivalent exact forms are accepted. Use the reasoning to check student work.

1. 45 L/min.

- The volume changes from 140 L to 320 L: a gain of 180 L.
 - Time changes by 4 min; rate = $\frac{180}{4} = 45$ L/min.
-

2. 20 units/min; it ignores the non-zero starting amount.

- Use changes: $\frac{195-75}{9-3} = \frac{120}{6} = 20$.
 - The model has 15 units already present at time zero.
-

3. -4°C/min .

- Temperature change = $58 - 82 = -24^{\circ}\text{C}$.
 - Time interval is 6 min, so rate = $\frac{-24}{6} = -4^{\circ}\text{C/min}$.
-

4. \$8/kg.

- Cost change = $88 - 48 = \$40$; mass change = $11 - 6 = 5$ kg.
 - Rate = $\frac{40}{5} = \$8/\text{kg}$.
-

5. 1.4 km/min; 84 km/h.

- $\frac{21}{15} = 1.4$ km/min.
 - $1.4 \times 60 = 84$ km/h.
-

6. Q is faster by 30 L/min.

- P delivers $2.5 \times 60 = 150$ L/min.
 - $180 - 150 = 30$ L/min.
-

6E / Worked answers

Teacher appendix. Equivalent exact forms are accepted. Use the reasoning to check student work.

7. 48 m/min; 3 min.

- $0.8 \times 60 = 48 \text{ m/min.}$
 - $\text{Time} = \frac{144}{48} = 3 \text{ min.}$
-

8. A: 12 units/min; B: 18 units/min. B is faster.

- A gains $180 - 60 = 120$ units in 10 min.
 - B gains 180 units in 10 min; compare rates 12 and 18.
-

9. 13 minutes.

- Required increase = $615 - 160 = 455 \text{ L.}$
 - $\text{Time} = \frac{455}{35} = 13 \text{ min.}$
-

10. 12 minutes from process start.

- Cooling rate = $\frac{46-70}{8-2} = -4^\circ\text{C/min.}$
 - From 46°C it needs $\frac{16}{4} = 4$ more min; $8 + 4 = 12$.
-

11. 12 minutes; 288 new units.

- New production required = $328 - 40 = 288$ units.
 - $\text{Time} = \frac{288}{24} = 12 \text{ min.}$
-

12. Pump B; 18 minutes.

- A needs $\frac{2700}{120} = 22.5$ min, too long. B gives 150 L/min and needs $\frac{2700}{150} = 18$ min.
 - C gives 200 L/min but exceeds the \$75 budget.
-

CHAPTER 6 / 6F

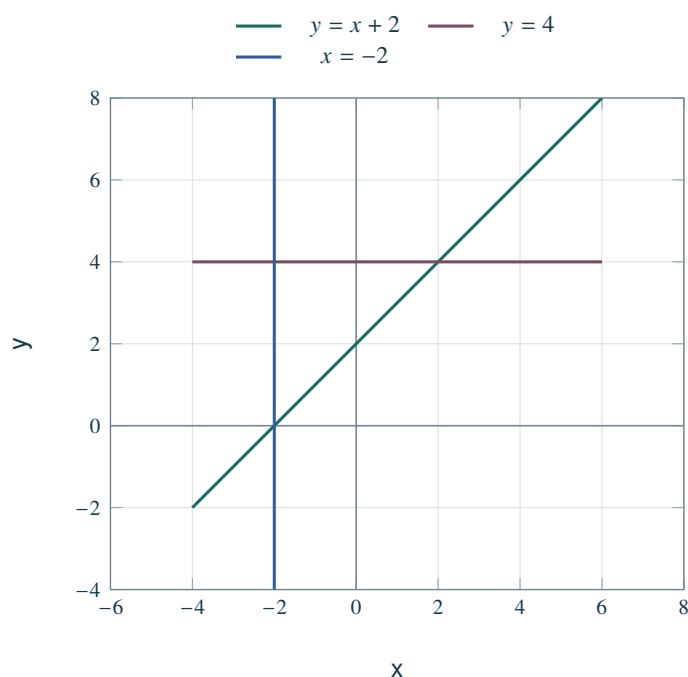
 $y = mx + c$ and special lines

Name: _____ Class: _____ Date: _____

Use the gradient and intercept to describe a line, sketch it and decide whether it fits a practical task. Distinguish sloping, horizontal and vertical lines.

What you will learn

- Identify m and c , including negative and fractional values.
- Build a line through a given point and check it by substitution.
- Recognise horizontal and vertical lines and their special equations.

See the mathematics

Compare a sloping line with horizontal and vertical special lines.

Key ideas

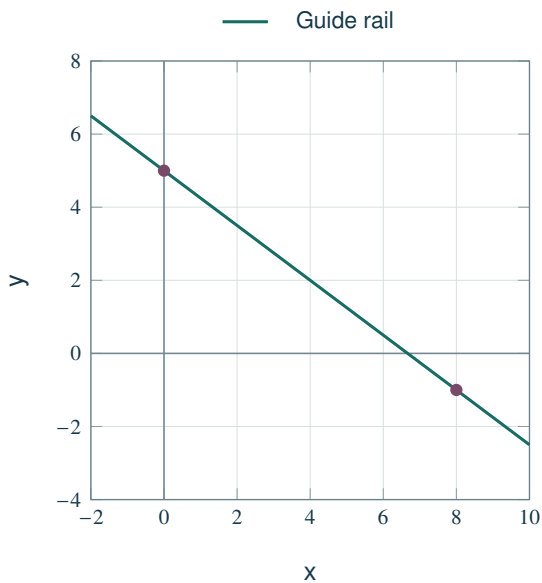
- In $y = mx + c$, m is the gradient and $(0, c)$ is the y -intercept.
- For $m = \frac{p}{q}$, a move of q units right changes y by p units. Keep the sign.
- A horizontal line has equation $y = k$ and gradient 0. A vertical line has equation $x = k$ and undefined gradient.
- A point lies on a line only if substituting both coordinates makes the equation true.

Pages 1–8: student resource. Pages 9–10: worked answers.

Worked examples / connect the calculation to the graph

1. Read a fractional gradient

A guide rail has equation $y = -\frac{3}{4}x + 5$. Identify the intercept and find the height at $x = 8$.



1. Compare with $y = mx + c$: $m = -\frac{3}{4}$ and $c = 5$.

2. Substitute $x = 8$: $y = -\frac{3}{4}(8) + 5 = -6 + 5$.

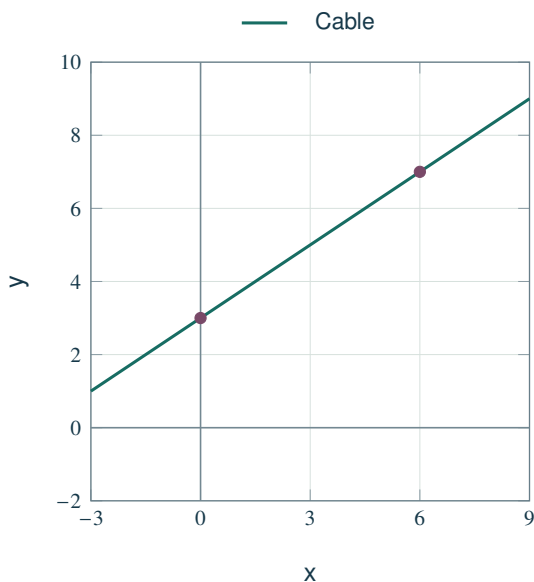
Result

The y -intercept is $(0, 5)$; the point at $x = 8$ is $(8, -1)$.

The worked model marks its intercept and the point found by substitution.

2. Build a line through a point

A cable has gradient $\frac{2}{3}$ and passes through $(6, 7)$. Find its equation.



1. Write $y = \frac{2}{3}x + c$.

2. Substitute $(6, 7)$: $7 = 4 + c$, so $c = 3$.

Result

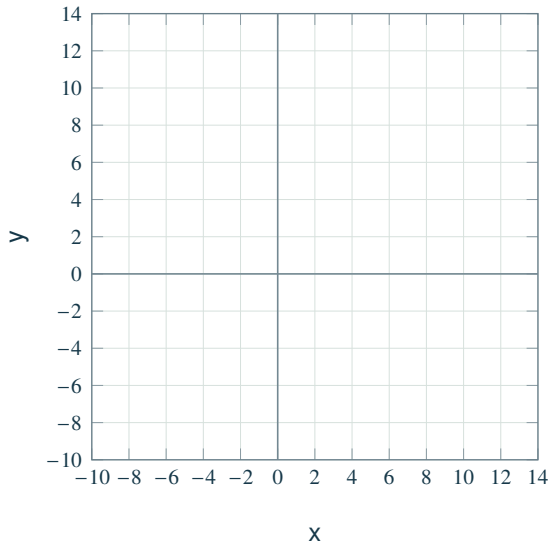
$y = \frac{2}{3}x + 3$.

The completed cable model passes through the required point and its calculated intercept.

1. Read and sketch

Use the graph to record your thinking. Show calculations and include units where needed.

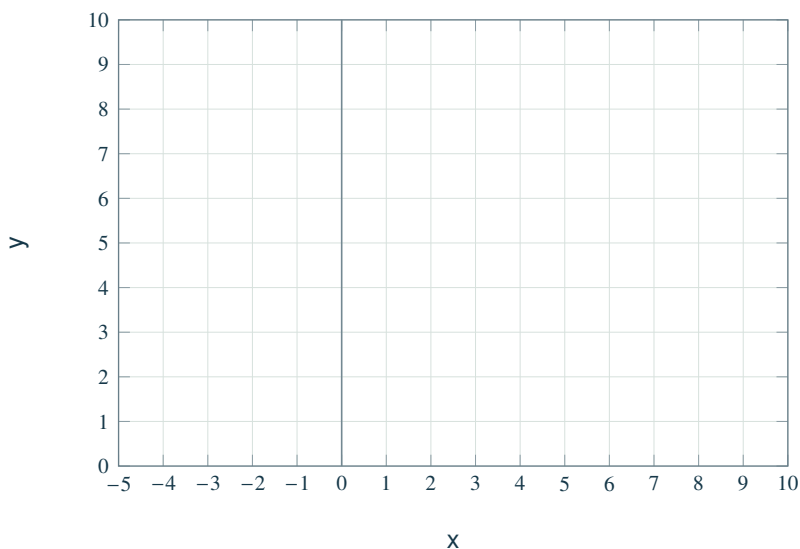
1. For $y = 4x - 7$, state the gradient and y -intercept. Find y when $x = 3$. Explain what a one-unit increase in x does to y . Use the grid to show the line and the point you calculate.



Construction grid for the line, intercept and calculated point.

WORKING / DECISION

2. For $y = -\frac{2}{5}x + 6$, state m and c . Starting at the y -intercept, give a second point reached by moving 5 units right. Mark both points on the grid and draw the line.



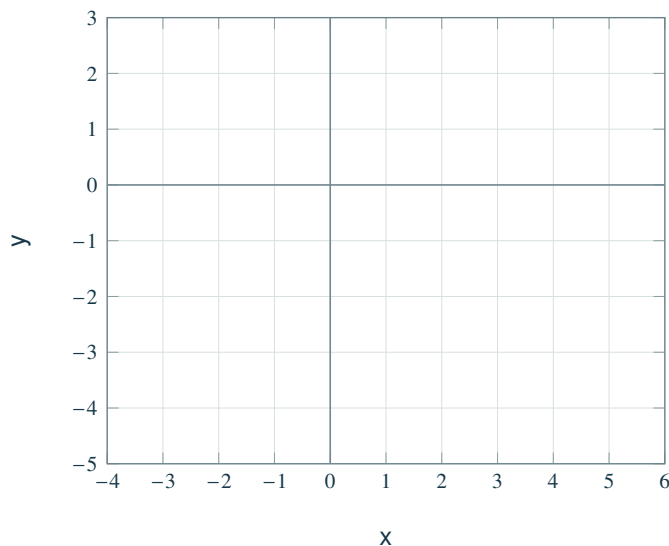
Construction grid for a fractional negative gradient.

WORKING / DECISION

1. Read and sketch

Use the graph to record your thinking. Show calculations and include units where needed.

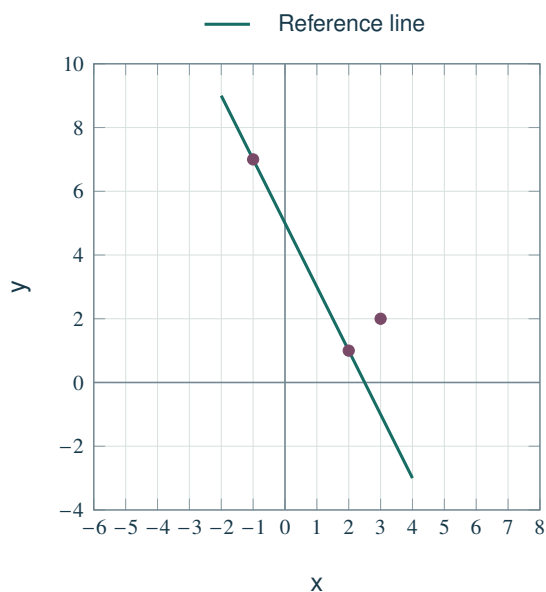
3. Sketch $y = \frac{1}{2}x - 2$ for $-4 \leq x \leq 6$. Mark the y -intercept and the point where the line crosses the x -axis.



WORKING / DECISION

Calculate the points before constructing the requested line segment.

4. Decide whether each point is on $y = -2x + 5$: $A(2, 1)$, $B(-1, 7)$, $C(3, 2)$. Show one substitution for each. Use the reference graph to check your decisions.



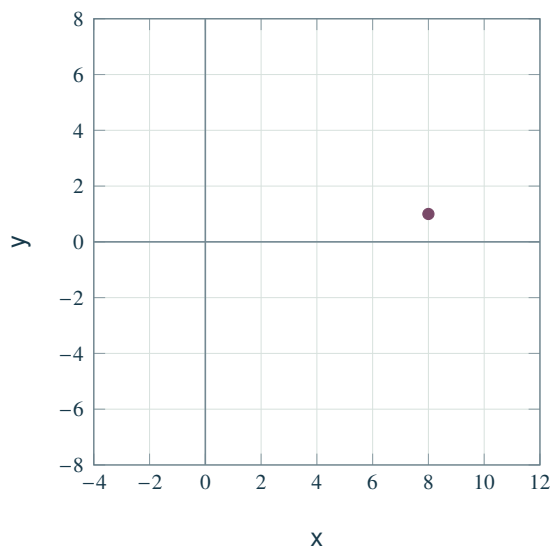
WORKING / DECISION

The supplied line and candidate points $A(2, 1)$, $B(-1, 7)$, $C(3, 2)$.

2. Design the equation

Use the graph to record your thinking. Show calculations and include units where needed.

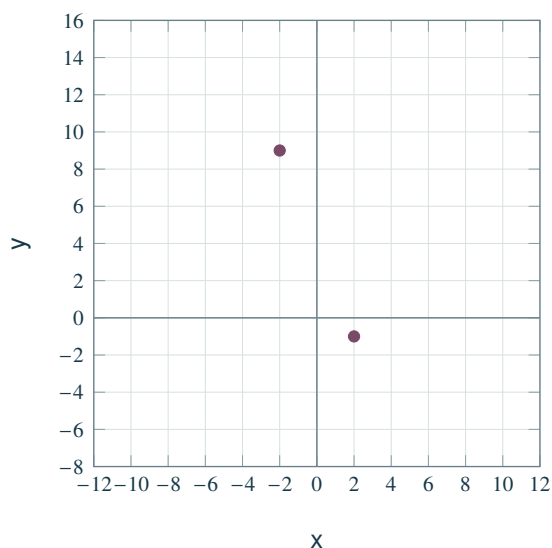
5. A survey line has gradient $\frac{3}{4}$ and must pass through $(8, 1)$. Find its equation and its y -intercept. Draw your proposed line through the marked point.



WORKING / DECISION

Only the required point is provided. Construct the survey line after finding its equation.

6. A ramp line has gradient $-\frac{5}{2}$ and passes through $(-2, 9)$. Find its equation. Does it pass through $(2, -1)$? Draw the line to support your check.



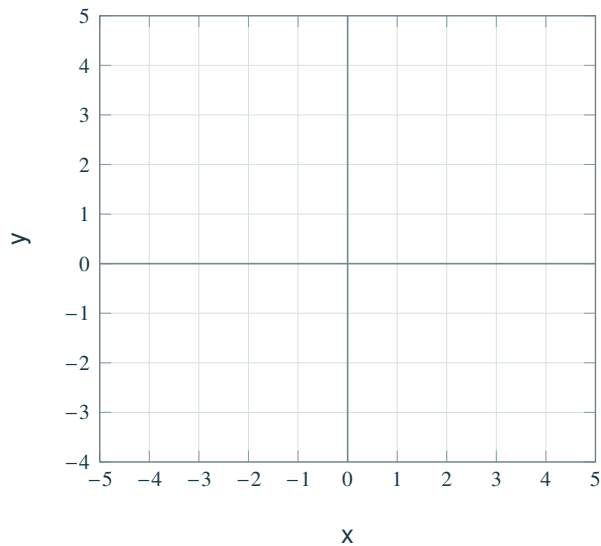
WORKING / DECISION

The required point and candidate check point are marked; the ramp remains for you to draw.

2. Design the equation

Use the graph to record your thinking. Show calculations and include units where needed.

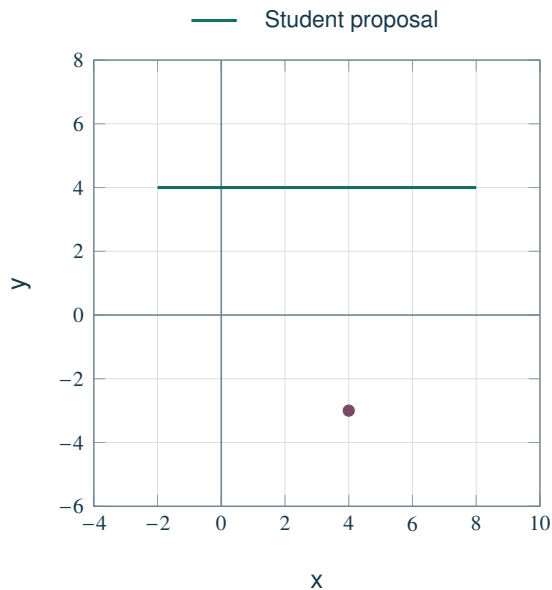
7. Draw $y = 3$ and $x = -2$ on the same grid. State each gradient and their intersection point.



Construction grid for both special lines.

WORKING / DECISION

8. A student says the vertical line through $(4, -3)$ is $y = 4$. Correct the equation, explain the mistake and state whether the line has a y -intercept. Draw the corrected line on the supplied diagram.



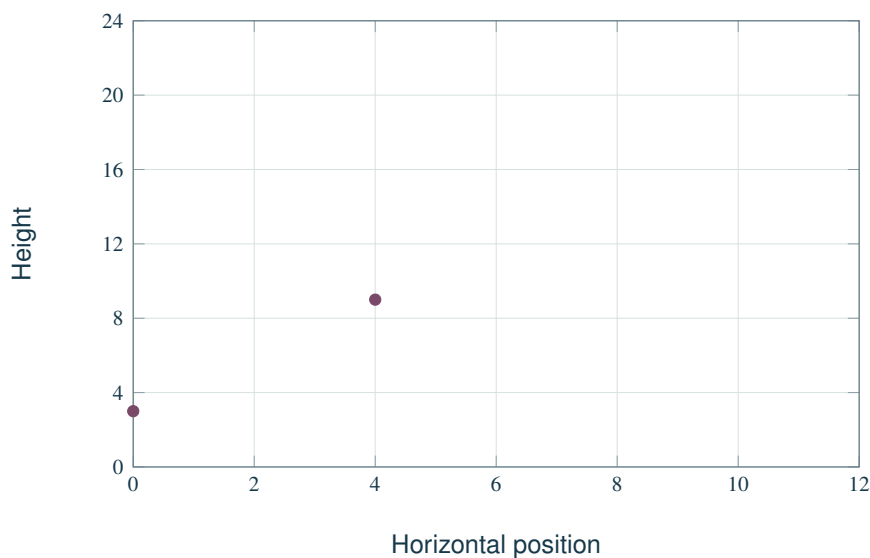
WORKING / DECISION

The student proposal and required point are given. Add your corrected line.

3. Apply and justify

Use the graph to record your thinking. Show calculations and include units where needed.

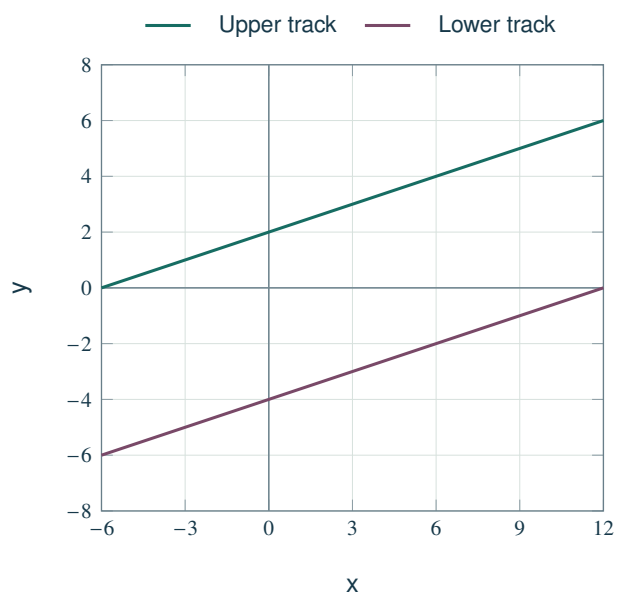
9. A sensor must show height 9 at horizontal position 4 and have a starting height of 3 at position 0. Find its linear rule and predict the height at position 10. Use the grid to show the two known readings and your predicted point.



WORKING / DECISION

The two known sensor readings are plotted. Add the recovered model and prediction.

10. Two tracks are $y = \frac{1}{3}x + 2$ and $y = \frac{1}{3}x - 4$. Compare their directions and find their vertical separation at $x = 9$. Is this the shortest distance between the tracks? Mark the vertical separation on the graph.



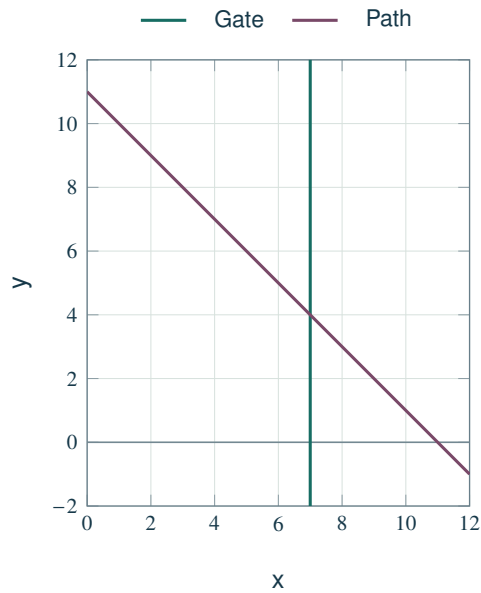
WORKING / DECISION

Both supplied tracks are shown. Mark the separation at the specified horizontal position.

3. Apply and justify

Use the graph to record your thinking. Show calculations and include units where needed.

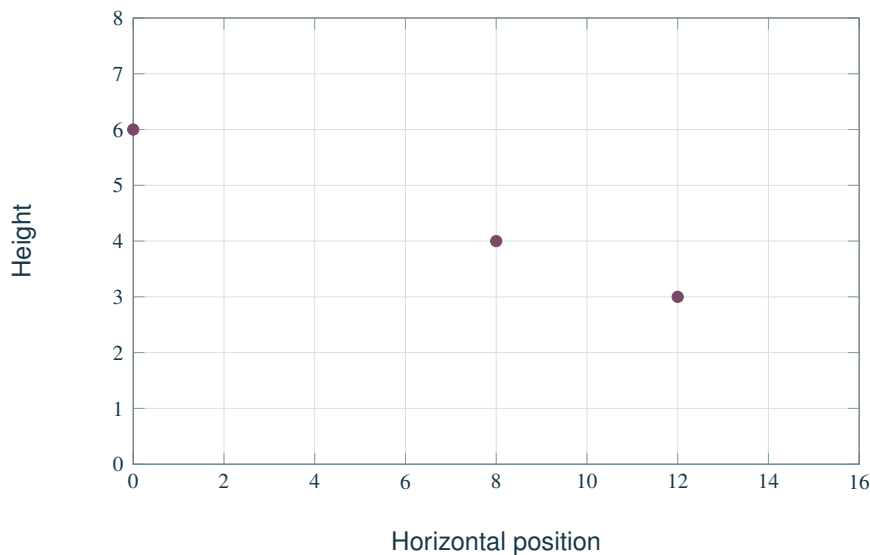
11. A gate is fixed at $x = 7$. A sloping path follows $y = -x + 11$. Find where the path meets the gate. State the gate's equation and why $y = mx + c$ cannot describe it. Mark the intersection on the graph.



WORKING / DECISION

The supplied gate and path; calculate and mark their meeting point.

12. A roof edge must pass through $(0, 6)$ and $(8, 4)$. A support at $x = 12$ reaches height 3. Does this single straight roof edge meet the support? Find and use its equation, then draw the roof on the diagram.



WORKING / DECISION

Two roof points and the top of the proposed support are supplied. Draw your roof model.

6F / Worked answers

Teacher appendix. Equivalent exact forms are accepted. Use the reasoning to check student work.

1. $m = 4$; intercept $(0, -7)$; $y = 5$; y rises by 4.

- m is the coefficient of x ; c is the constant term.
 - At $x = 3$, $y = 12 - 7 = 5$.
-

2. $m = -\frac{2}{5}$, $c = 6$; second point $(5, 4)$.

- A run of 5 produces a rise of -2 .
 - Start at $(0, 6)$, then move to $(5, 4)$.
-

3. Intercepts $(0, -2)$ and $(4, 0)$.

- At $x = -4$, $y = -4$; at $x = 6$, $y = 1$.
 - Draw a straight line through these points and both intercepts.
-

4. A and B lie on the line; C does not.

- A: $-2(2) + 5 = 1$. B: $-2(-1) + 5 = 7$.
 - C: $-2(3) + 5 = -1$, not 2.
-

5. $y = \frac{3}{4}x - 5$; y -intercept $(0, -5)$.

- $1 = \frac{3}{4}(8) + c = 6 + c$.
 - Therefore $c = -5$.
-

6. $y = -\frac{5}{2}x + 4$; yes.

- $9 = -\frac{5}{2}(-2) + c = 5 + c$, so $c = 4$.
 - At $x = 2$, $y = -5 + 4 = -1$.
-

6F / Worked answers

Teacher appendix. Equivalent exact forms are accepted. Use the reasoning to check student work.

7. Gradients 0 and undefined; intersection $(-2, 3)$.

- $y = 3$ is horizontal. $x = -2$ is vertical.
 - Their intersection satisfies both equations.
-

8. $x = 4$; no y -intercept.

- Every point on a vertical line has the same x -coordinate.
 - $y = 4$ would be horizontal. The line $x = 4$ never reaches $x = 0$.
-

9. $y = \frac{3}{2}x + 3$; height 18.

- $m = \frac{9-3}{4-0} = \frac{6}{4} = \frac{3}{2}$.
 - At $x = 10$, $y = 15 + 3 = 18$.
-

10. Same gradient; vertical separation 6; not the shortest distance.

- At $x = 9$ the heights are 5 and -1 , a vertical difference of 6.
 - The shortest distance between sloping parallel lines is measured perpendicular to them.
-

11. Intersection $(7, 4)$; gate $x = 7$.

- Set $x = 7$: $y = -7 + 11 = 4$.
 - A vertical line has multiple y -values for the same x and undefined gradient.
-

12. Yes; $y = -\frac{1}{4}x + 6$ gives $y = 3$ at $x = 12$.

- $m = \frac{4-6}{8-0} = -\frac{1}{4}$, with $c = 6$.
 - Substitution gives $-\frac{1}{4}(12) + 6 = 3$.
-

CHAPTER 6 / 6G

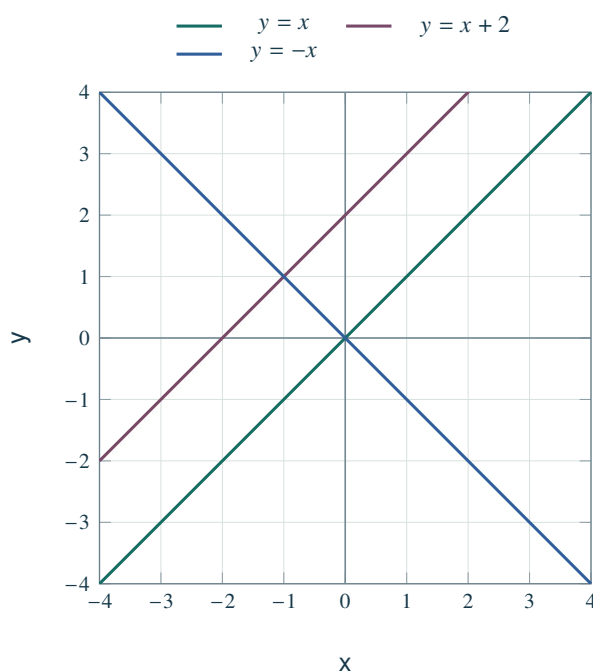
Parallel lines and perpendicular lines

Name: _____ Class: _____ Date: _____

Use gradients to plan parallel routes and right-angle crossings. Check the direction and the required point separately before approving a line.

What you will learn

- Find gradients from equations and pairs of points.
- Construct parallel and perpendicular lines through required points.
- Explain negative reciprocals and horizontal/vertical exceptions.

See the mathematics

Equal-scale axes preserve the visual right angle. The two rising lines are parallel.

Key ideas

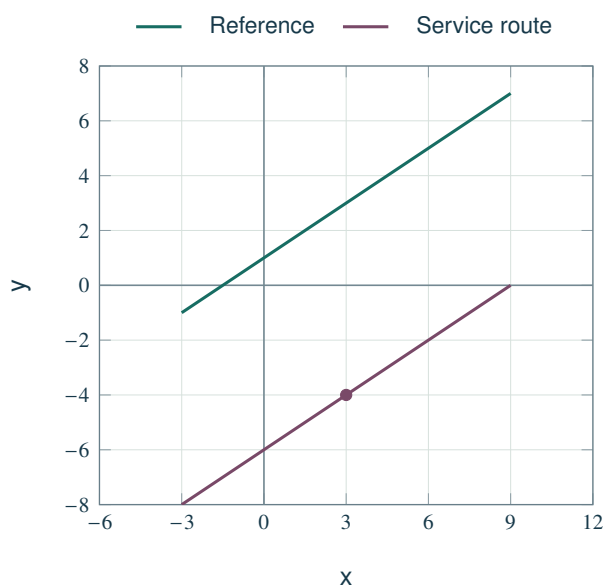
- Distinct parallel non-vertical lines have the same gradient and different intercepts.
- For two non-zero finite gradients, perpendicular lines satisfy $m_1 m_2 = -1$.
- The negative reciprocal of $\frac{p}{q}$ is $-\frac{q}{p}$; change both the sign and the fraction orientation.
- A horizontal line is perpendicular to a vertical line. Do not use division by zero in the reciprocal rule.

Pages 1–8: student resource. Pages 9–10: worked answers.

Worked examples / connect the calculation to the graph

1. A parallel service route

Find the line parallel to $y = \frac{2}{3}x + 1$ that passes through $(3, -4)$.



Reference road and the completed parallel service route.

1. Keep $m = \frac{2}{3}$ and use $y = \frac{2}{3}x + c$.

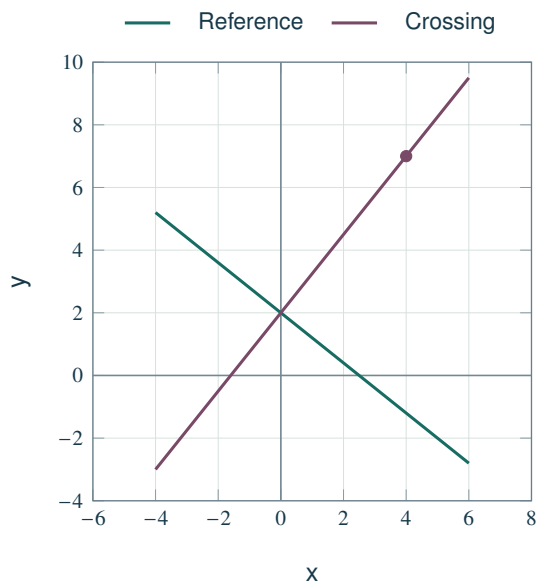
2. $-4 = 2 + c$, so $c = -6$.

Result

$$y = \frac{2}{3}x - 6.$$

2. A right-angle crossing

Find the line perpendicular to $y = -\frac{4}{5}x + 2$ through $(4, 7)$.



Completed crossing on equal-scale axes; both lines and the required point are shown.

1. The negative reciprocal of $-\frac{4}{5}$ is $\frac{5}{4}$.

2. $7 = \frac{5}{4}(4) + c = 5 + c$, so $c = 2$.

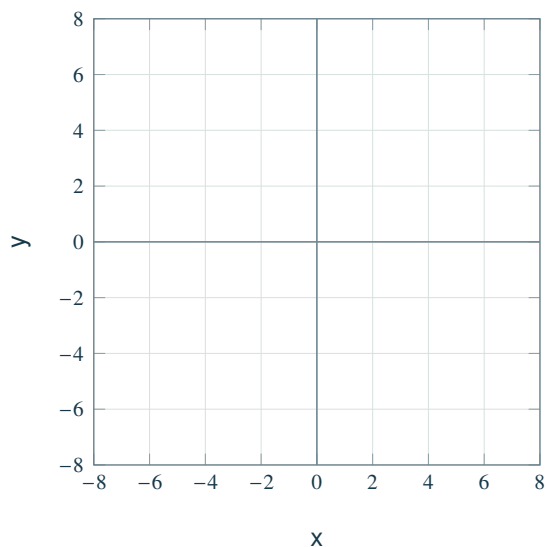
Result

$$y = \frac{5}{4}x + 2; \left(-\frac{4}{5}\right)\left(\frac{5}{4}\right) = -1.$$

1. Compare gradients

Use the graph to record your thinking. Show calculations and include units where needed.

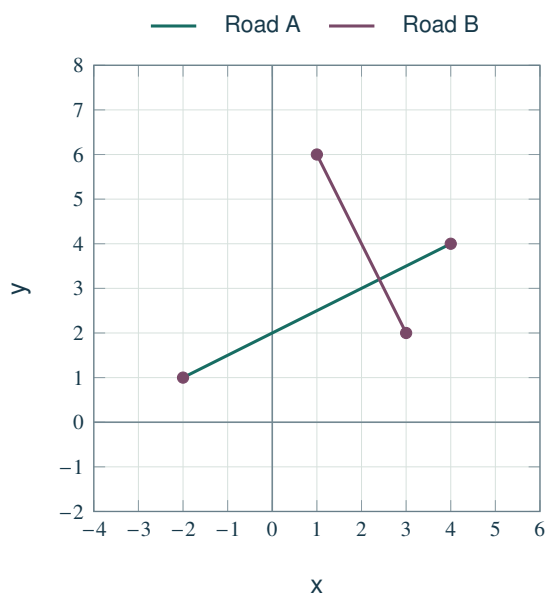
1. Classify each gradient pair as parallel, perpendicular or neither: (a) $\frac{3}{5}$ and $\frac{3}{5}$; (b) $-\frac{2}{7}$ and $\frac{7}{2}$; (c) 4 and -4 . Use the grid to sketch and label an example for each pair.



WORKING / DECISION

Construction grid for labelled examples of the three gradient pairs.

2. Road A passes through $(-2, 1)$ and $(4, 4)$. Road B passes through $(1, 6)$ and $(3, 2)$. Decide whether their directions are perpendicular. Use the equal-scale graph to check your calculation.



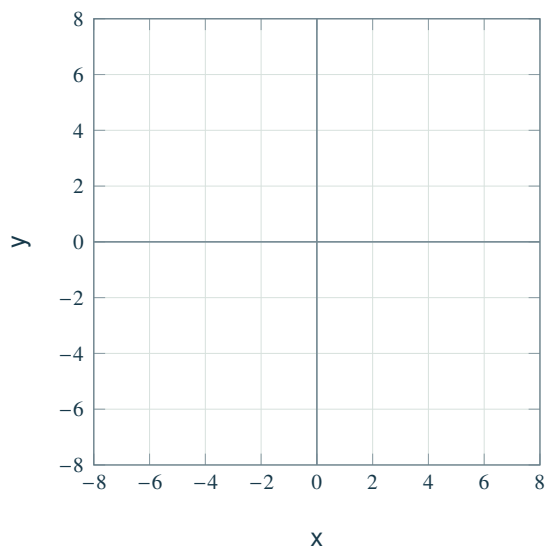
WORKING / DECISION

A joins its two supplied points; B joins its own supplied points. Both axes use the same unit scale.

1. Compare gradients

Use the graph to record your thinking. Show calculations and include units where needed.

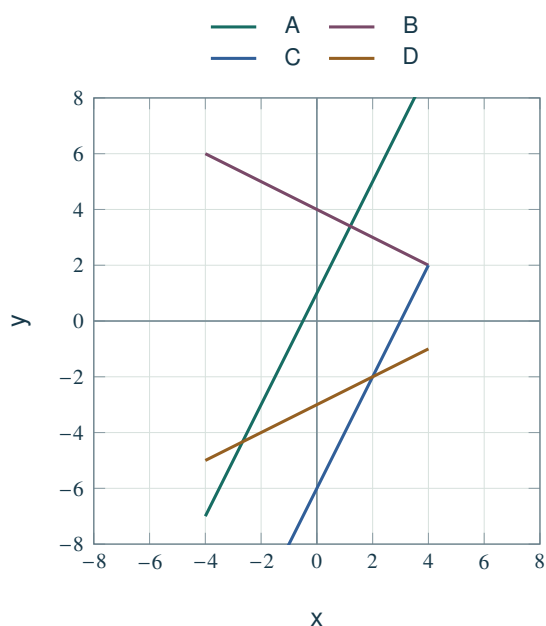
3. Write the perpendicular gradient for each: (a) $\frac{5}{8}$; (b) -3 ; (c) -0.25 . Check each answer by multiplication. On the grid, sketch each supplied direction and its proposed perpendicular through the origin.



WORKING / DECISION

Construction grid: label each direction pair a, b or c.

4. Lines A, B, C and D have equations $y = 2x + 1$, $y = -\frac{1}{2}x + 4$, $y = 2x - 6$ and $y = \frac{1}{2}x - 3$. List every parallel pair and every perpendicular pair. Match your decisions to the labelled graph.



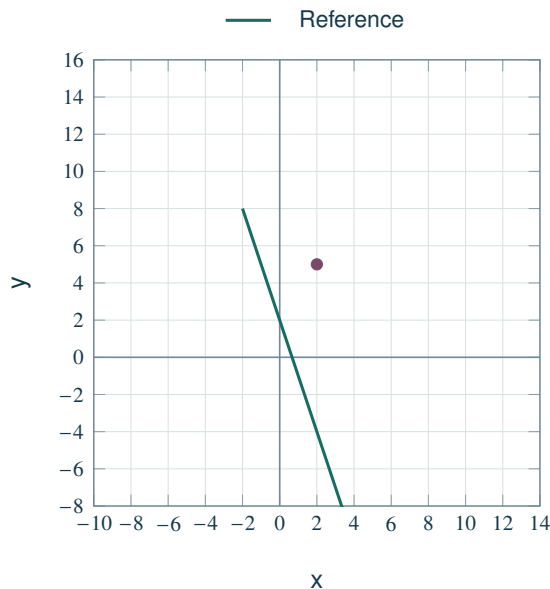
WORKING / DECISION

A, B, C and D match the equations in the stated order.

2. Build the routes

Use the graph to record your thinking. Show calculations and include units where needed.

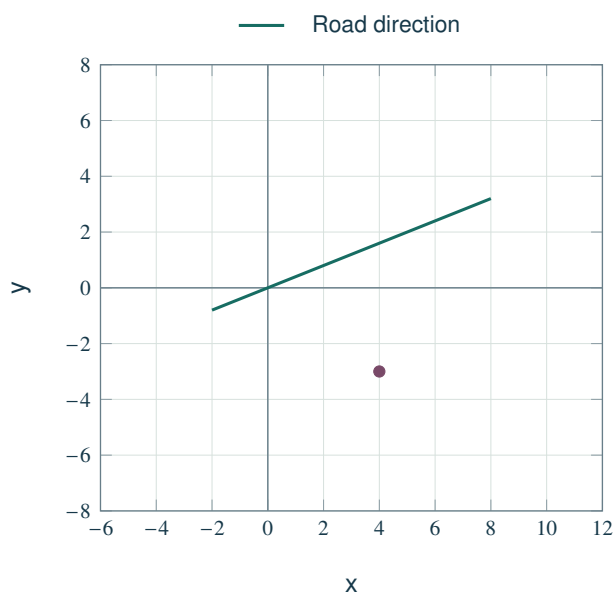
5. Find the equation of a line parallel to $y = -3x + 2$ through $(2, 5)$. Verify that your line passes through the point, then construct it on the diagram.



WORKING / DECISION

The reference line and required point are given. Add the new parallel line.

6. An emergency track must meet a road of gradient $\frac{2}{5}$ at a right angle and pass through $(4, -3)$. Find the emergency track equation and draw it on the diagram.



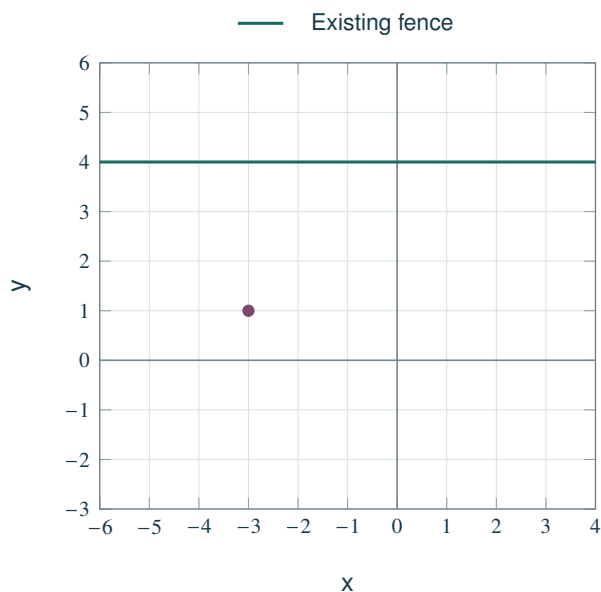
WORKING / DECISION

The supplied road direction and the required point. Construct the emergency track.

2. Build the routes

Use the graph to record your thinking. Show calculations and include units where needed.

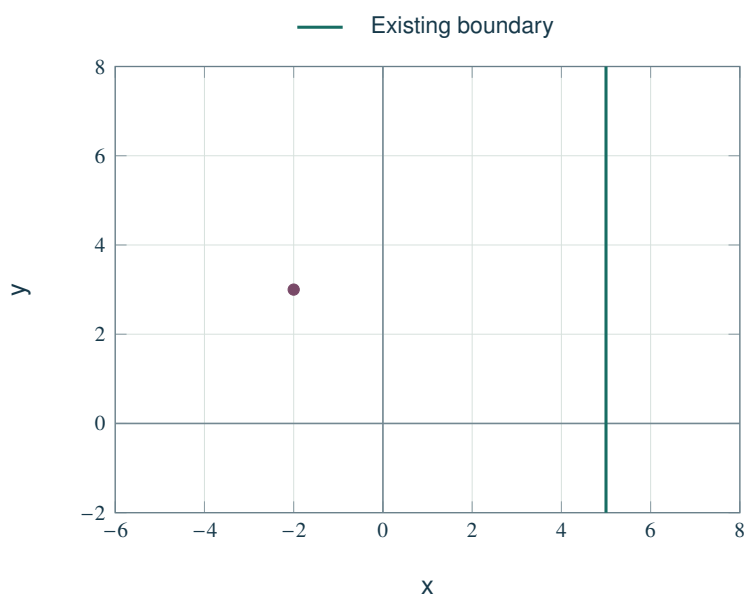
7. A fence follows $y = 4$. Draw the perpendicular fence through $(-3, 1)$. State its equation and explain why no finite m works.



WORKING / DECISION

The existing horizontal fence and required point; add the new fence.

8. A boundary follows $x = 5$. Give both a parallel boundary and a perpendicular crossing through $(-2, 3)$. Draw both new lines on the supplied diagram.



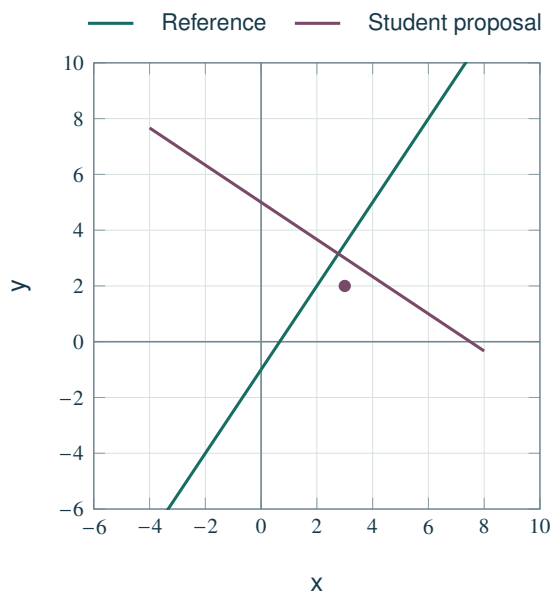
WORKING / DECISION

The existing vertical boundary and required point are supplied.

3. Check the engineering

Use the graph to record your thinking. Show calculations and include units where needed.

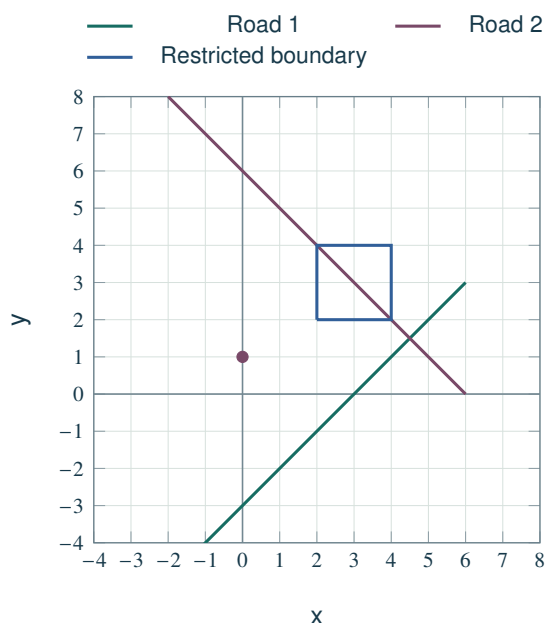
9. A student proposes $y = -\frac{2}{3}x + 5$ as a perpendicular to $y = \frac{3}{2}x - 1$ through $(3, 2)$. Check the gradient and point separately. Repair the equation if needed and show the repaired line on the graph.



WORKING / DECISION

The reference, the student proposal and the required point are supplied. The repaired line is not drawn.

10. A route through $(0, 1)$ may be parallel to either road $y = x - 3$ or road $y = -x + 6$. The entire square $2 \leq x \leq 4$, $2 \leq y \leq 4$ is restricted, including its boundary. Which proposed route avoids the square? Draw your route.



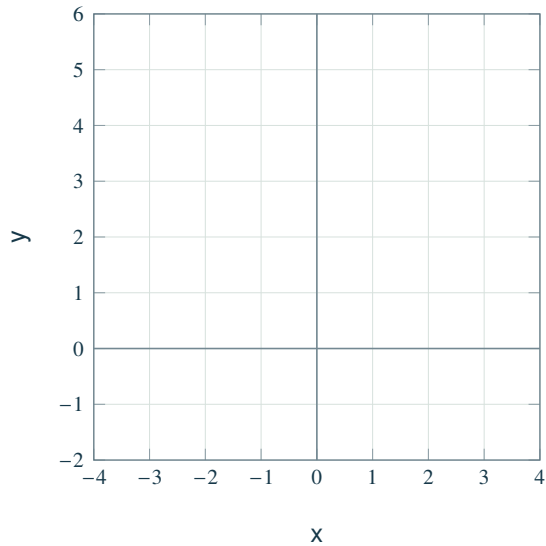
WORKING / DECISION

The closed square is restricted, including its edge. Existing roads are shown; construct the candidate routes through the marked point.

3. Check the engineering

Use the graph to record your thinking. Show calculations and include units where needed.

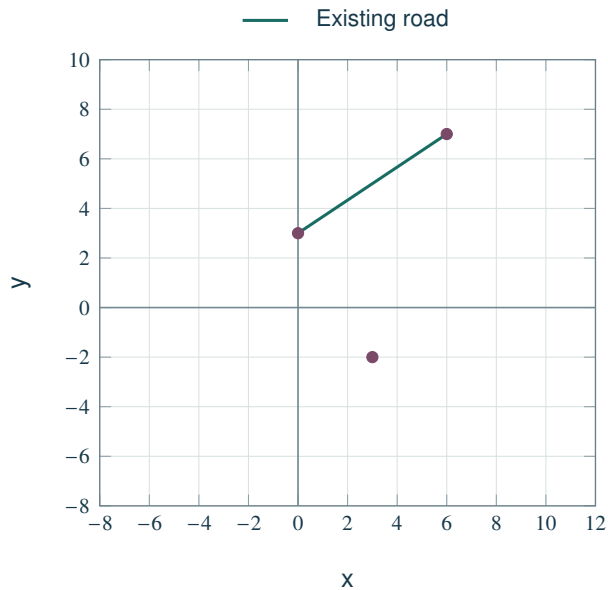
11. Sketch $y = x + 2$ and $y = -x + 2$ on equal-scale axes. Find their intersection and verify their angle using gradients.



Construct both lines and label their intersection.

WORKING / DECISION

12. A road passes through $(0, 3)$ and $(6, 7)$. Build a parallel service line through $(3, -2)$, then a perpendicular rescue line through the same point. Give both equations and one check for each, then draw the two new lines.



The existing road and the shared required point. Add both service and rescue routes.

WORKING / DECISION

6G / Worked answers

Teacher appendix. Equivalent exact forms are accepted. Use the reasoning to check student work.

1. (a) Parallel directions; (b) perpendicular; (c) neither.

- Equal gradients give parallel directions; distinct lines also need different intercepts.
 - $\left(-\frac{2}{7}\right)\left(\frac{7}{2}\right) = -1$; $4(-4) = -16$.
-

2. Yes: $m_A = \frac{1}{2}$ and $m_B = -2$.

- $m_A = \frac{4-1}{4-(-2)} = \frac{3}{6}$.
 - $m_B = \frac{2-6}{3-1} = -2$; their product is -1 .
-

3. (a) $-\frac{8}{5}$; (b) $\frac{1}{3}$; (c) 4.

- Change the sign and take the reciprocal.
 - $\frac{5}{8}\left(-\frac{8}{5}\right) = -1$, $(-3)\frac{1}{3} = -1$, $\left(-\frac{1}{4}\right)4 = -1$.
-

4. Parallel: A,C. Perpendicular: A,B and B,C.

- The gradients are $2, -\frac{1}{2}, 2, \frac{1}{2}$.
 - Only the pairs listed have equal gradients or product -1 .
-

5. $y = -3x + 11$.

- $5 = -3(2) + c$, so $c = 11$.
 - At $x = 2$, $-6 + 11 = 5$.
-

6. $y = -\frac{5}{2}x + 7$.

- Required gradient is $-\frac{5}{2}$.
 - $-3 = -\frac{5}{2}(4) + c = -10 + c$, so $c = 7$.
-

6G / Worked answers

Teacher appendix. Equivalent exact forms are accepted. Use the reasoning to check student work.

7. $x = -3$, a vertical line with undefined gradient.

- A right angle to a horizontal line is vertical.
 - Its horizontal run is zero, so $\frac{\text{rise}}{\text{run}}$ is undefined.
-

8. Parallel: $x = -2$. Perpendicular: $y = 3$.

- A parallel vertical line keeps x constant at -2 .
 - The perpendicular line is horizontal with y always 3.
-

9. Gradient is correct; intercept should be 4. $y = -\frac{2}{3}x + 4$.

- $\frac{3}{2} \left(-\frac{2}{3}\right) = -1$.
 - The proposed line gives $y = 3$ at $x = 3$. Use $2 = -2 + c$ to get $c = 4$.
-

10. Choose $y = -x + 1$.

- The alternatives through $(0, 1)$ are $y = x + 1$ and $y = -x + 1$.
 - $y = x + 1$ enters the square; for $2 \leq x \leq 4$, $-x + 1$ is between -3 and -1 , below it.
-

11. Intersection $(0, 2)$; the lines are perpendicular.

- $x + 2 = -x + 2$ gives $2x = 0$.
 - Gradients 1 and -1 have product -1 .
-

12. Service: $y = \frac{2}{3}x - 4$. Rescue: $y = -\frac{3}{2}x + \frac{5}{2}$.

- Road gradient $= \frac{4}{6} = \frac{2}{3}$; $-2 = 2 + c$ gives service $c = -4$.
 - Perpendicular $m = -\frac{3}{2}$; $-2 = -\frac{9}{2} + c$ gives $c = \frac{5}{2}$. Both give $y = -2$ at $x = 3$.
-

CHAPTER 6 / 6H

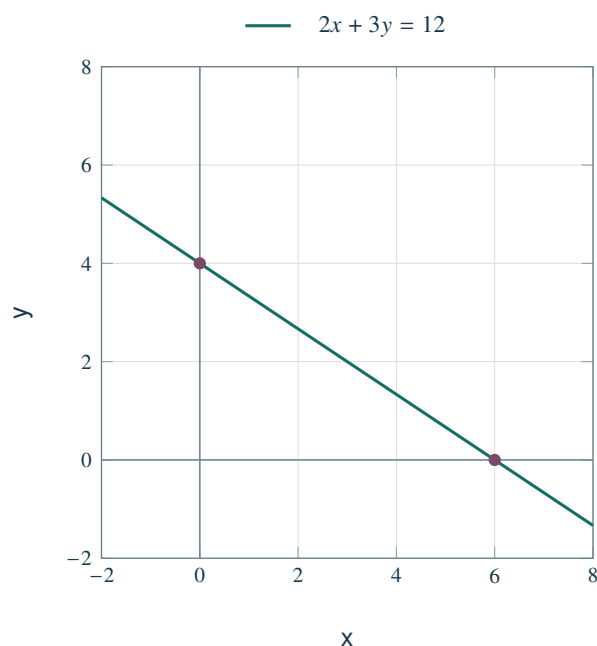
Graphing straight lines using intercepts

Name: _____ Class: _____ Date: _____

Find where a line crosses each axis and use those points to construct the graph. Recognise when the intercept method needs another point.

What you will learn

- Find x - and y -intercepts by setting the other variable to zero.
- Sketch lines from intercepts, including fractional and negative values.
- Interpret intercepts in a context and handle special cases.

See the mathematics

The line is fixed by two different axis intercepts. Each intercept has one zero coordinate.

Key ideas

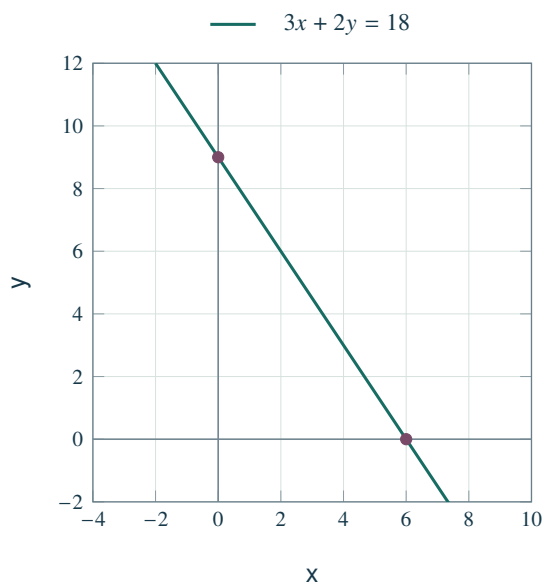
- At an x -intercept $y = 0$; at a y -intercept $x = 0$.
- For $ax + by = d$ with $a \neq 0$ and $b \neq 0$, the intercepts are $\left(\frac{d}{a}, 0\right)$ and $\left(0, \frac{d}{b}\right)$.
- Two different points determine a straight line. If both intercepts are $(0, 0)$, find another point.
- A horizontal or vertical line may miss an axis. Only the non-negative part of a graph may make sense in a physical context.

Pages 1–8: student resource. Pages 9–10: worked answers.

Worked examples / connect the calculation to the graph

1. Construct from two intercepts

Find the intercepts of $3x + 2y = 18$.



1. Set $y = 0$: $3x = 18$, giving $x = 6$.

2. Set $x = 0$: $2y = 18$, giving $y = 9$.

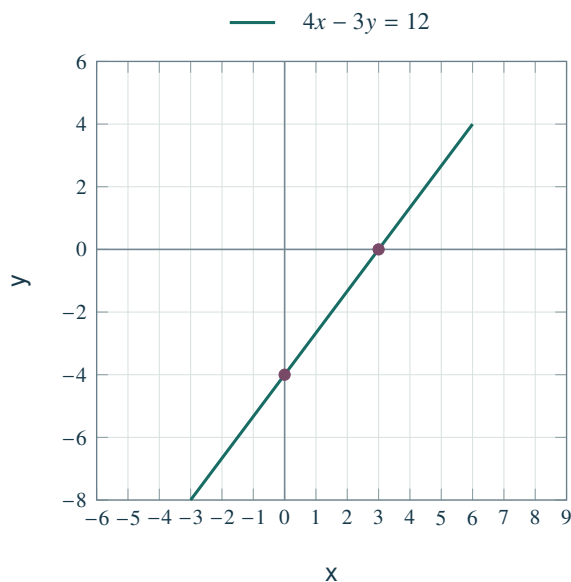
Result

Plot $(6, 0)$ and $(0, 9)$, then draw the straight line through them.

The worked example marks both calculated intercepts and the resulting straight line.

2. Work with negative values

Find the intercepts of $4x - 3y = 12$.



1. With $y = 0$, $4x = 12$, so $x = 3$.

2. With $x = 0$, $-3y = 12$, so $y = -4$.

Result

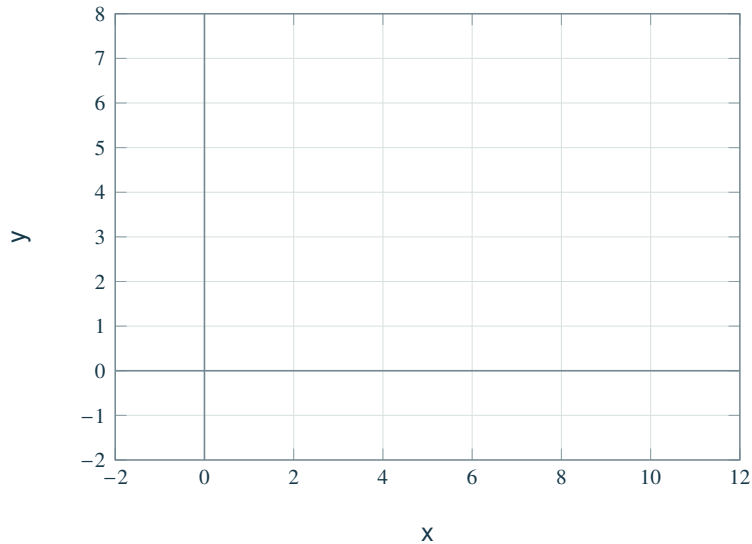
The intercepts are $(3, 0)$ and $(0, -4)$.

One intercept lies on the negative half of its axis in this worked example.

1. Find the crossing points

Use the graph to record your thinking. Show calculations and include units where needed.

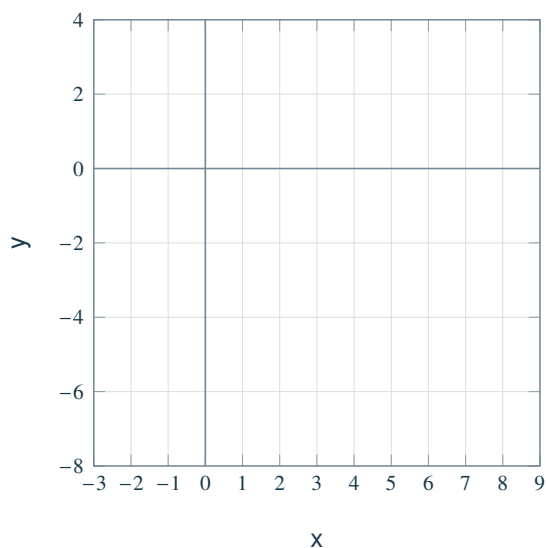
1. Find both intercepts of $2x + 5y = 20$. Explain why the y -coordinate of the x -intercept must be zero. Plot your intercepts and draw the line.



WORKING / DECISION

Construction grid: calculate and plot the two intercepts before joining them.

2. Find both intercepts of $3x - 2y = 12$. In which half of the y -axis does the line cross? Draw the line through your calculated intercepts.



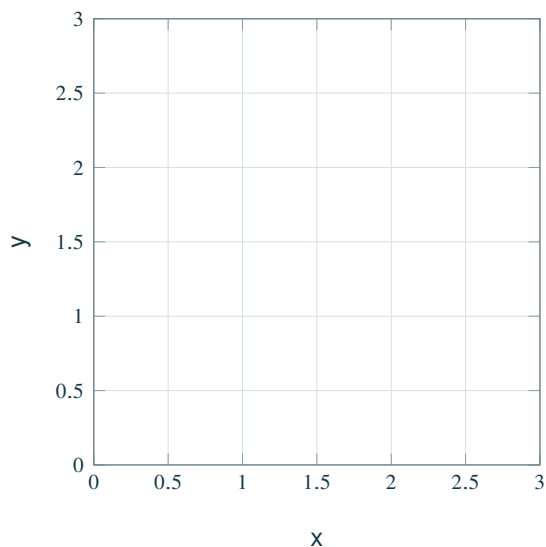
WORKING / DECISION

Construction grid with positive and negative axis values.

1. Find the crossing points

Use the graph to record your thinking. Show calculations and include units where needed.

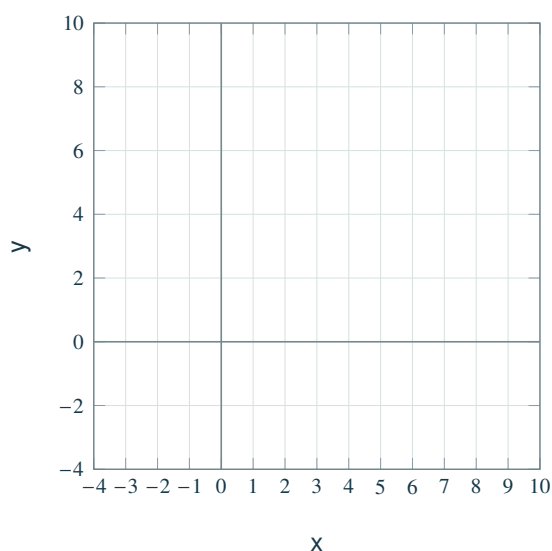
3. Find both intercepts of $4x + 6y = 9$. Leave answers as exact fractions. Mark the intercepts accurately on the half-unit grid and draw the line.



WORKING / DECISION

Construction grid: some required coordinates lie between marked half-unit ticks.

4. Find both intercepts of $y = -\frac{3}{2}x + 6$. Check the x -intercept in the original equation, then draw the line.



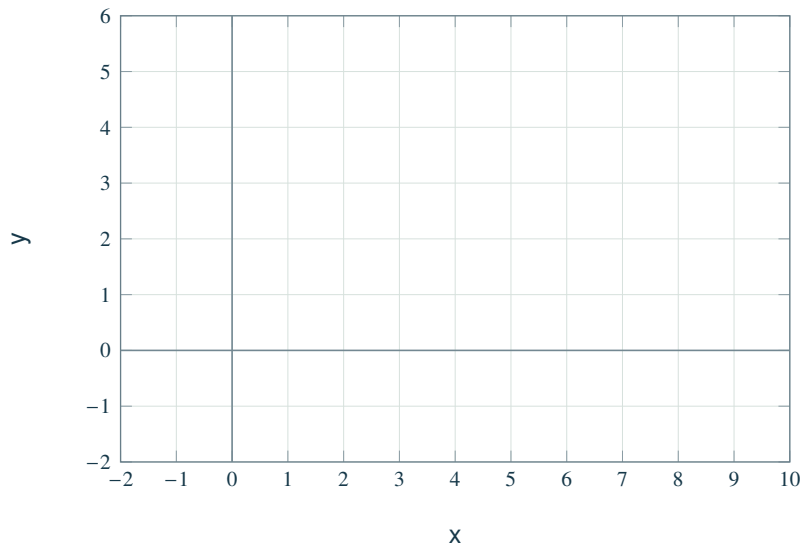
WORKING / DECISION

Construction grid for intercepts obtained from gradient-intercept form.

2. Plot and handle exceptions

Use the graph to record your thinking. Show calculations and include units where needed.

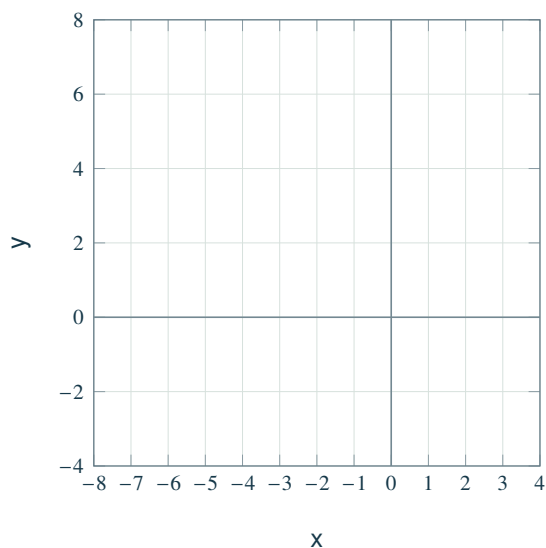
5. Use intercepts to sketch $x + 2y = 8$. State where the line crosses each axis and label the line.



WORKING / DECISION

Construction grid: mark and label both intercepts.

6. Use intercepts to sketch $2x - y = -6$. Give a third point to check the graph.



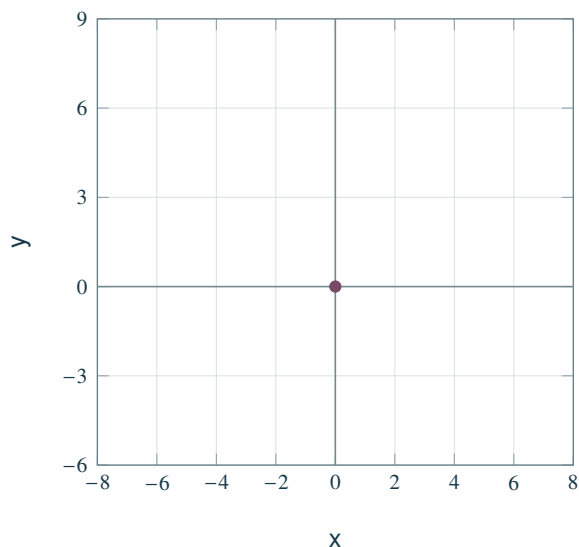
WORKING / DECISION

Construction grid for both intercepts and an independently checked third point.

2. Plot and handle exceptions

Use the graph to record your thinking. Show calculations and include units where needed.

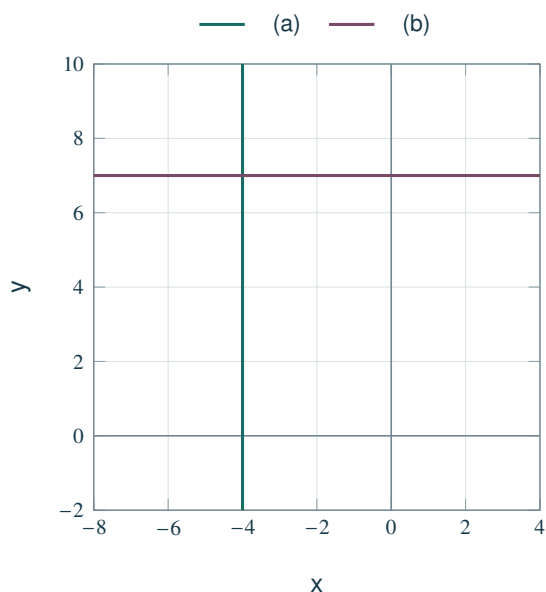
7. A student finds both intercepts of $y = 3x$ to be $(0, 0)$ and says a unique line can now be drawn. Explain what is missing and give a useful second point. Add your second point and the line to the diagram.



WORKING / DECISION

Only the shared intercept is provided. Add enough information to determine the line.

8. State the intercepts, if any, of (a) $x = -4$ and (b) $y = 7$. Explain why each line misses one axis. Mark the existing intercepts on the reference graph.



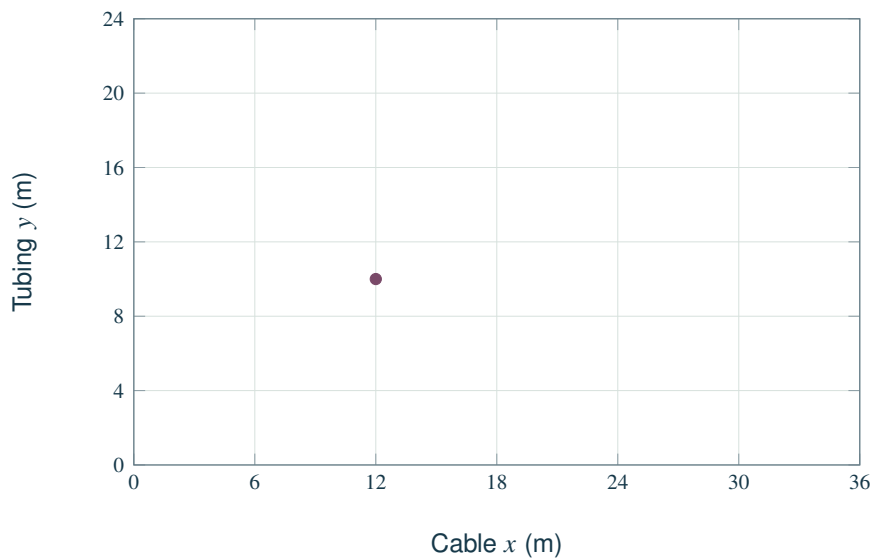
WORKING / DECISION

Reference graph of the two supplied special lines.

3. Use the intercepts

Use the graph to record your thinking. Show calculations and include units where needed.

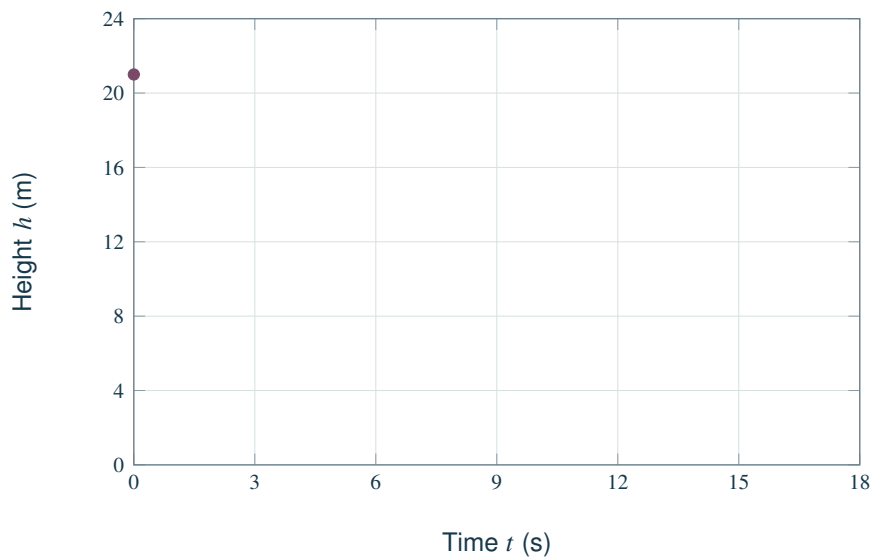
9. A workshop has \$180 for cable at \$6 per metre and tubing at \$9 per metre. Its full-budget boundary is $6x + 9y = 180$. Find and interpret both intercepts. Is (12, 10) affordable? Draw the boundary and show the proposed purchase.



WORKING / DECISION

The proposed purchase is marked. Construct the full-budget boundary from your intercepts.

10. A lift's height is $h = 21 - 1.5t$, with h in metres and t in seconds. Find the two intercepts and explain each one. Give the time interval before the lift reaches ground level and draw that part of the journey.



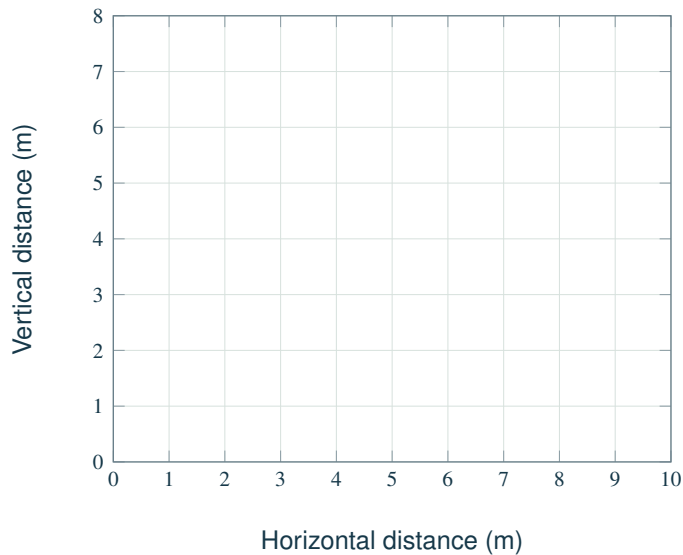
WORKING / DECISION

The starting height is given. Construct the journey only over its physical time interval.

3. Use the intercepts

Use the graph to record your thinking. Show calculations and include units where needed.

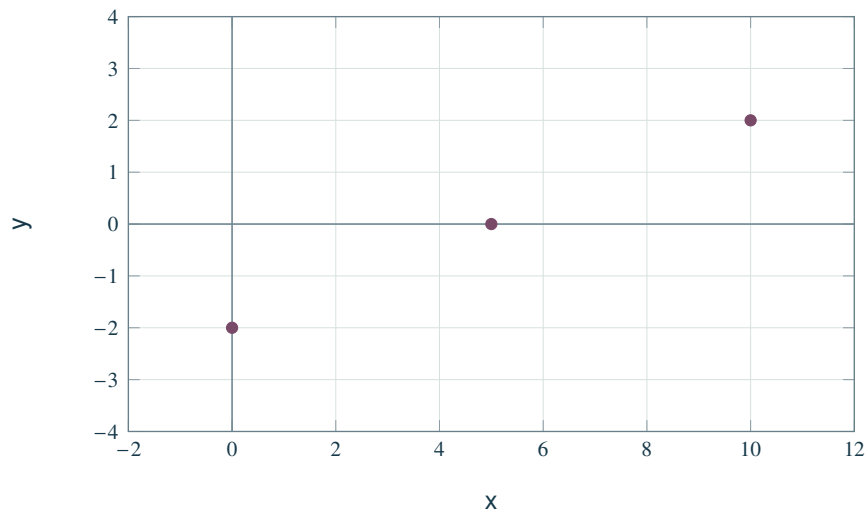
11. A triangular work zone has boundaries $x = 0$, $y = 0$ and $3x + 4y = 24$, with distances in metres. Find the two non-origin vertices and the area. Draw and shade the work zone on the grid.



WORKING / DECISION

Construction grid: use the axes and your calculated boundary to form the work zone.

12. A navigation boundary must pass through $(5, 0)$ and $(0, -2)$. Write its equation in the form $ax + by = d$ with integer coefficients, then check whether $(10, 2)$ lies on it. Draw the boundary through the supplied intercepts.



WORKING / DECISION

The required intercepts and proposed check point are supplied. Add the boundary.

6H / Worked answers

Teacher appendix. Equivalent exact forms are accepted. Use the reasoning to check student work.

1. x -intercept $(10, 0)$; y -intercept $(0, 4)$.

- $y = 0$ gives $2x = 20$. $x = 0$ gives $5y = 20$.
 - Every point on the x -axis has y -coordinate 0.
-

2. $(4, 0)$ and $(0, -6)$; negative half of the y -axis.

- Set $y = 0$: $x = \frac{12}{3} = 4$.
 - Set $x = 0$: $y = \frac{12}{-2} = -6$.
-

3. $(\frac{9}{4}, 0)$ and $(0, \frac{3}{2})$.

- At $y = 0$, $x = \frac{9}{4}$.
 - At $x = 0$, $y = \frac{9}{6} = \frac{3}{2}$.
-

4. $(4, 0)$ and $(0, 6)$.

- $0 = -\frac{3}{2}x + 6$ gives $x = 4$.
 - Substituting gives $-\frac{3}{2}(4) + 6 = 0$.
-

5. Straight line through $(8, 0)$ and $(0, 4)$.

- $y = 0$ gives $x = 8$; $x = 0$ gives $y = 4$.
 - Join and extend through both points.
-

6. Line through $(-3, 0)$, $(0, 6)$; another point is $(-2, 2)$.

- $y = 0$ gives $x = -3$; $x = 0$ gives $y = 6$.
 - At $x = -2$, $-4 - y = -6$ gives $y = 2$.
-

6H / Worked answers

Teacher appendix. Equivalent exact forms are accepted. Use the reasoning to check student work.

7. The two intercepts are the same point; use another point such as $(1, 3)$.

- A single point cannot determine the direction of a line.
 - Set $x = 1$ to obtain $y = 3$, then join $(0, 0)$ and $(1, 3)$.
-

8. (a) $(-4, 0)$, no y -intercept. (b) $(0, 7)$, no x -intercept.

- $x = -4$ can never have $x = 0$.
 - $y = 7$ can never have $y = 0$.
-

9. $(30, 0)$: all cable; $(0, 20)$: all tubing. $(12, 10)$ costs \$162, so is affordable.

- $\frac{180}{6} = 30$ and $\frac{180}{9} = 20$.
 - $6(12) + 9(10) = 72 + 90 = 162 \leq 180$.
-

10. $(0, 21)$: initial height; $(14, 0)$: reaches ground; $0 \leq t \leq 14$.

- At $t = 0$, $h = 21$.
 - Set $h = 0$: $1.5t = 21$, so $t = 14$.
-

11. Vertices $(8, 0)$, $(0, 6)$; area 24 m^2 .

- The intercept lengths are $\frac{24}{3} = 8$ and $\frac{24}{4} = 6$.
 - Area = $\frac{1}{2} \times 8 \times 6 = 24$.
-

12. $2x - 5y = 10$; $(10, 2)$ lies on it.

- Gradient = $\frac{0 - (-2)}{5 - 0} = \frac{2}{5}$, so $y = \frac{2}{5}x - 2$.
 - Multiply by 5 and rearrange. At $(10, 2)$, $20 - 10 = 10$.
-

CHAPTER 6 / 61

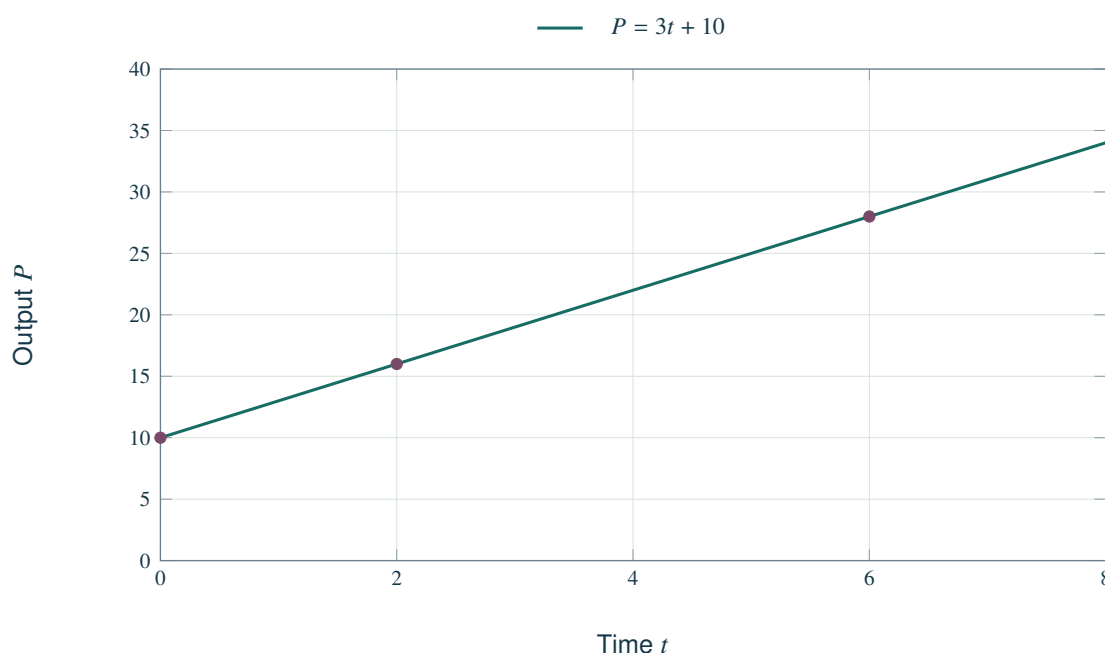
Linear modelling

Name: _____ Class: _____ Date: _____

Build a model from measured data, interpret its parameters and use it to make a practical decision. Check units and decide where a linear model remains sensible.

What you will learn

- Find a linear rule from two readings or a practical description.
- Interpret gradient and intercept using units and context.
- Use and compare models, including working backwards from a target.

See the mathematics

The starting output and rate have different roles in a linear model.

Key ideas

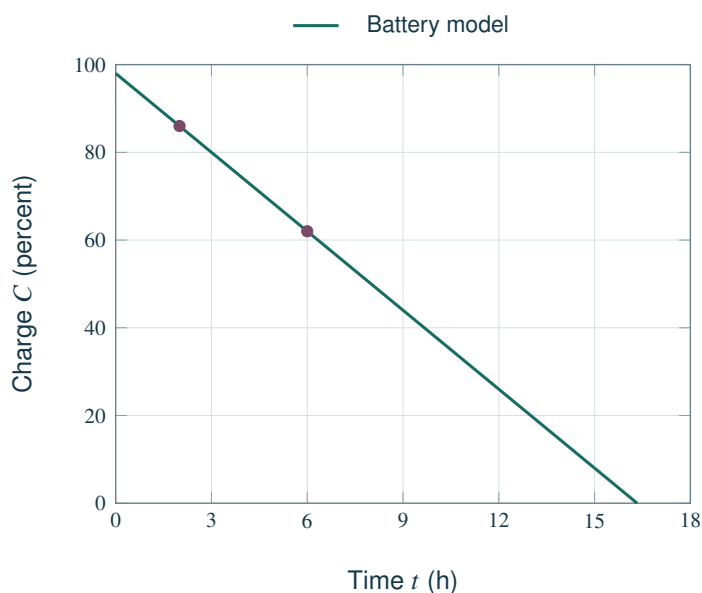
- For a non-vertical line, $m = \frac{y_2 - y_1}{x_2 - x_1}$. Keep the coordinate order consistent.
- Substitute a known point into $y = mx + c$ to find c . Check the second point.
- In context, the gradient is the change in output per unit of input; the intercept is the output when input is zero.
- A model is an assumption. Restrict its domain when negative time, volume or item counts would be impossible.

Pages 1–8: student resource. Pages 9–10: worked answers.

Worked examples / connect the calculation to the graph

1. Recover a battery model

A battery reports 86% charge after 2 hours and 62% after 6 hours. Model its percentage charge C after t hours.



$$1. m = \frac{62-86}{6-2} = -6 \text{ percentage points per hour.}$$

$$2. 86 = -6(2) + c, \text{ so } c = 98.$$

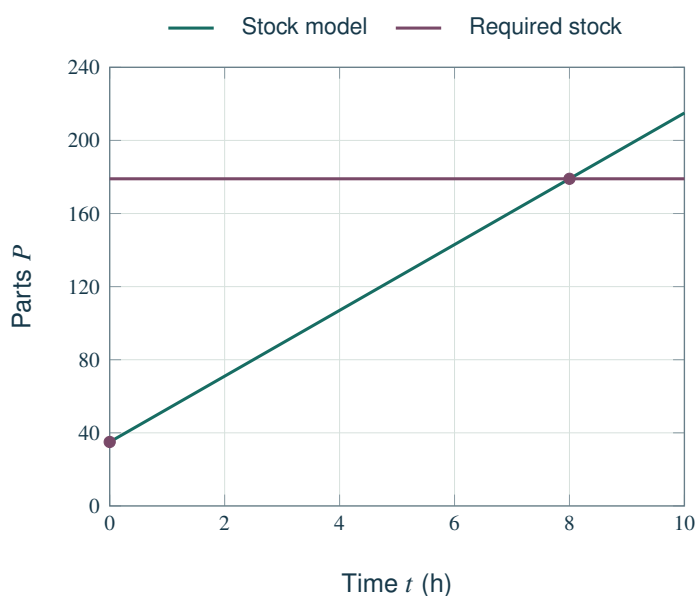
Result

$C = -6t + 98$, until the battery reaches 0%.

The complete worked battery model includes both observed readings and its physically meaningful endpoint.

2. Use a model backwards

A workshop already has 35 completed parts and produces 18 parts per hour. When will the stock reach 179 parts?



$$1. \text{ The stock model is } P = 18t + 35.$$

$$2. 179 = 18t + 35 \text{ gives } 144 = 18t.$$

Result

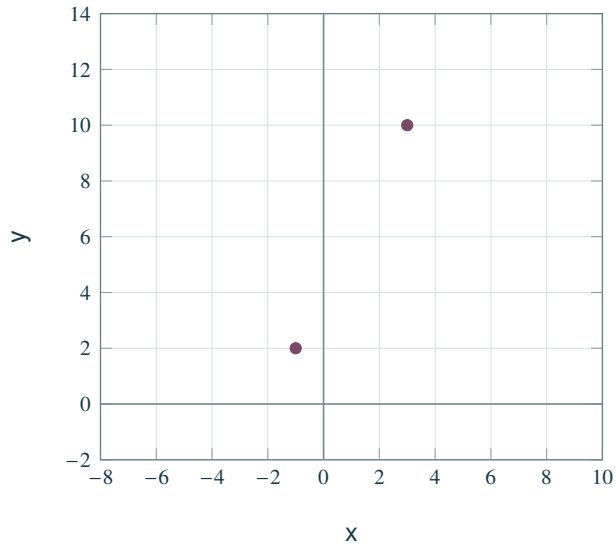
$t = 8$ hours.

The worked example uses the model backwards from the required stock level.

1. Recover the model

Use the graph to record your thinking. Show calculations and include units where needed.

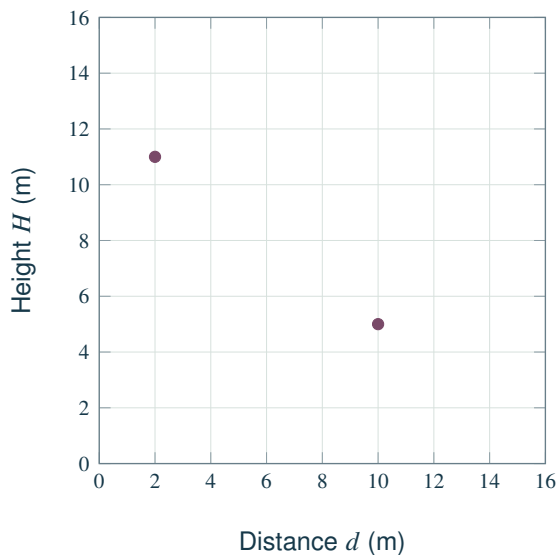
1. A straight route passes through $(-1, 2)$ and $(3, 10)$. Find its equation and check both points. Draw the route on the diagram.



WORKING / DECISION

Only the two supplied route points are shown. Construct the recovered line.

2. A cable height is 11 m at horizontal distance 2 m and 5 m at distance 10 m. Find the height model $H = md + c$. Plot the model through the two recorded points.



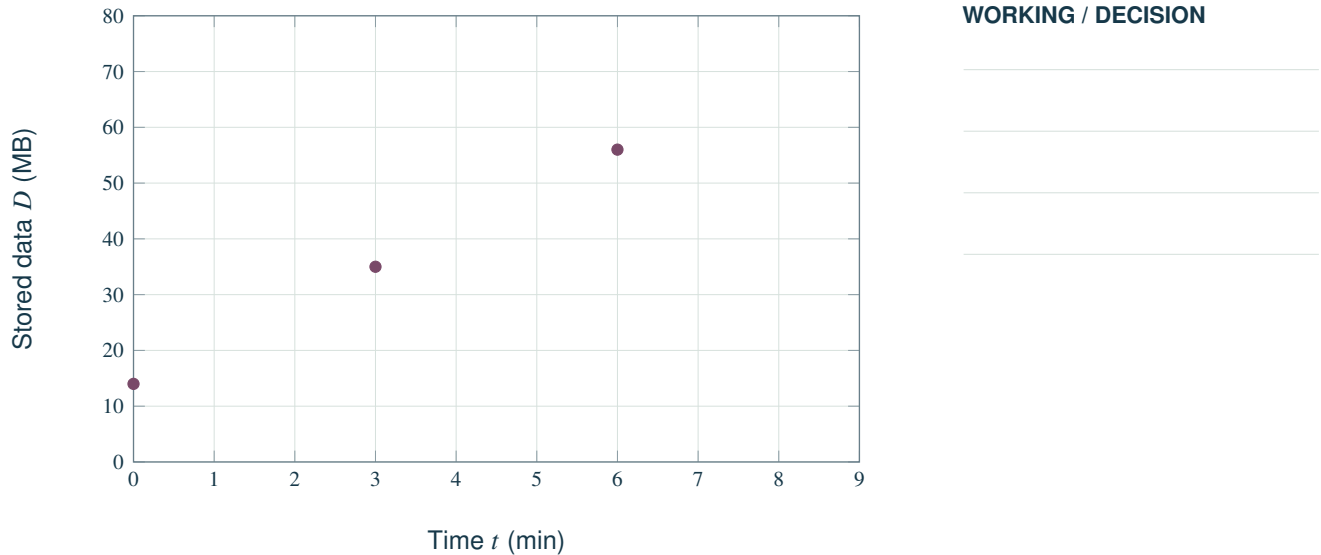
WORKING / DECISION

Measured cable heights. Add the model and label its starting-height prediction.

1. Recover the model

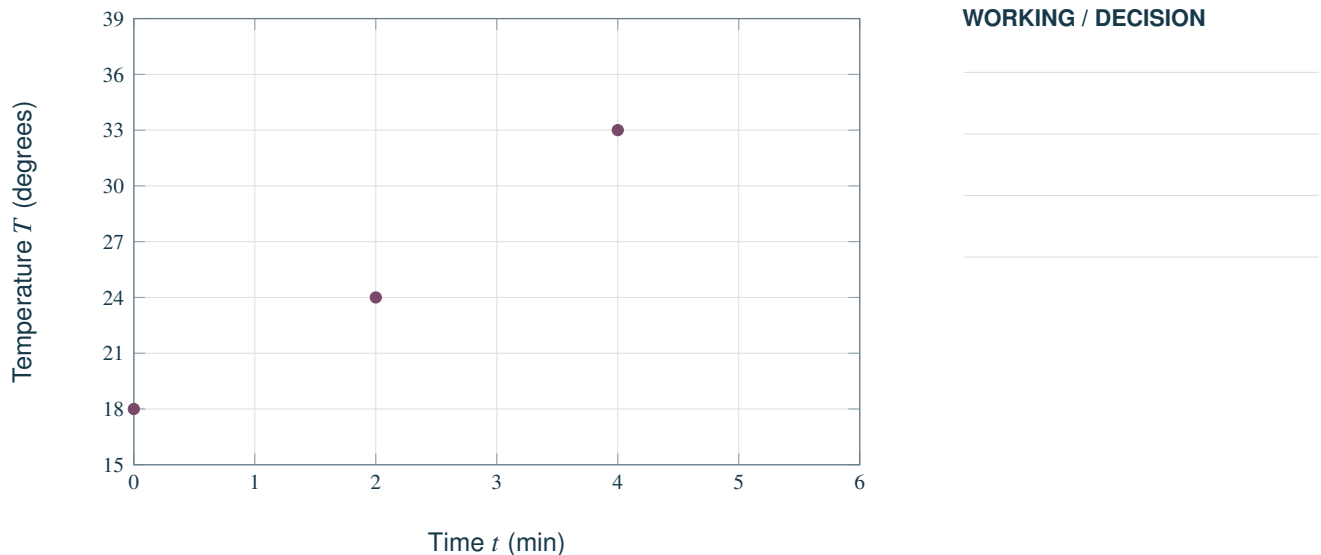
Use the graph to record your thinking. Show calculations and include units where needed.

3. A scanner gives these readings: time $t = 0, 3, 6$ minutes; stored data $D = 14, 35, 56$ MB. Decide whether a linear model fits and write it. Draw your proposed model through the graph of the readings.



All three supplied scanner readings are shown; use them to assess a single linear model.

4. A sensor gives (t, T) readings $(0, 18), (2, 24), (4, 33)$. Can one straight-line model fit all three exactly? Show the two interval rates and compare them with the plotted points.

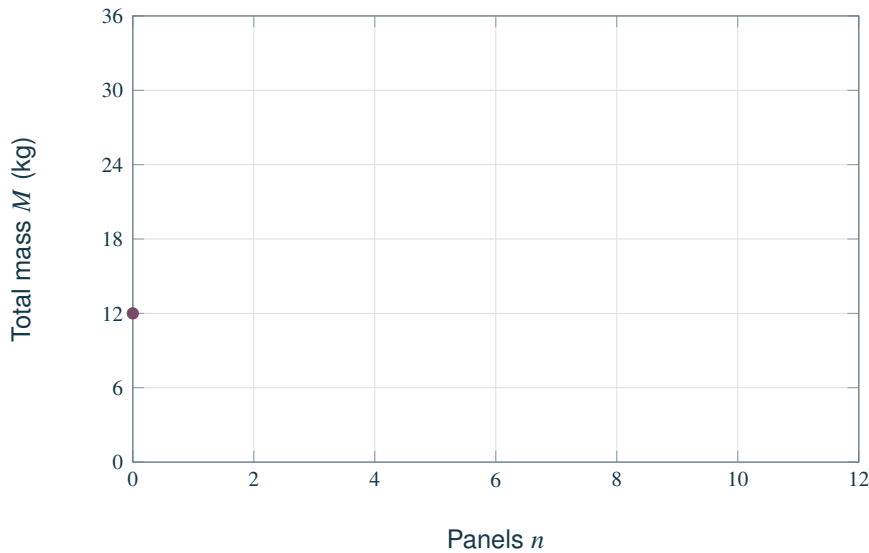


The graph contains only the recorded temperatures, not a fitted line.

2. Use the parameters

Use the graph to record your thinking. Show calculations and include units where needed.

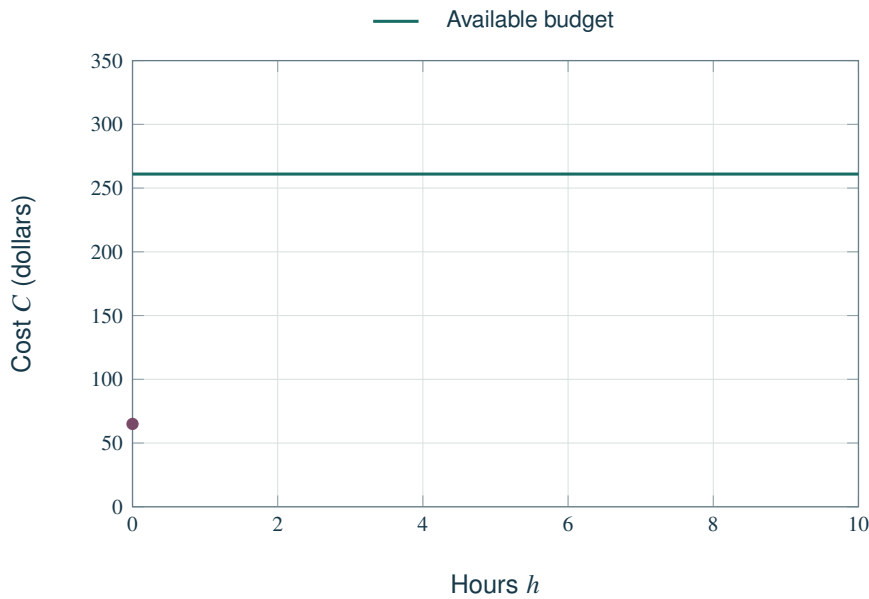
5. A display frame weighs 12 kg and each panel adds 1.75 kg. Write total mass M for n panels. Interpret both parameters and find the mass with 8 panels. Graph the model and mark the eight-panel configuration.



WORKING / DECISION

The empty frame is shown. Add the mass model and the requested configuration.

6. A maker charges a \$65 setup fee and \$28 per hour. Write C in terms of h . A customer has \$261. How many hours can they buy? Construct the cost model and show the purchase on the graph.



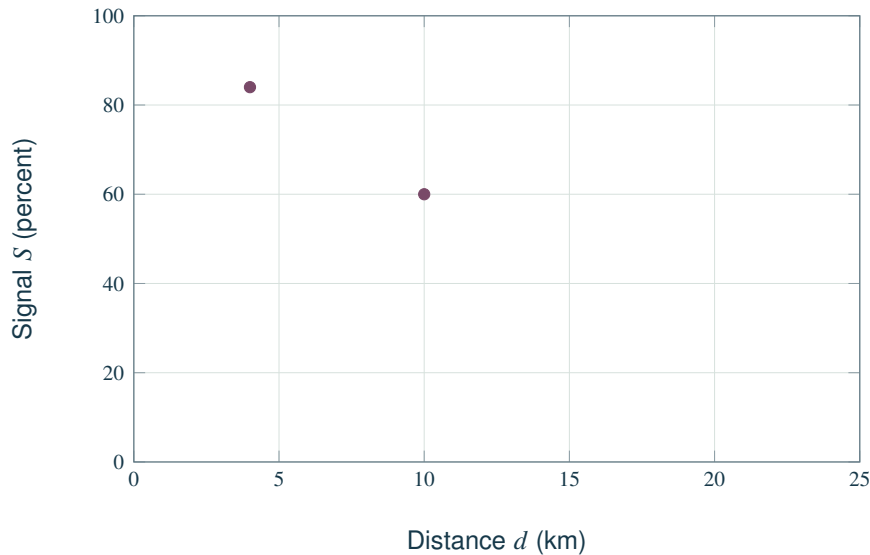
WORKING / DECISION

The setup fee and available budget are supplied. Add the maker cost model.

2. Use the parameters

Use the graph to record your thinking. Show calculations and include units where needed.

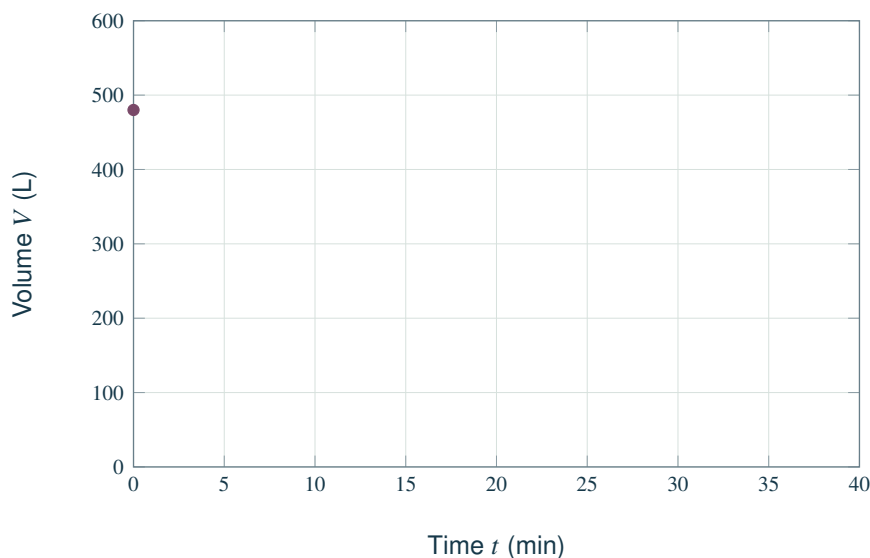
7. A rover's signal is 84% at 4 km and 60% at 10 km. Find $S = md + c$ using percentage units. Predict the percentage signal at 15 km and add that prediction to the graph of the supplied readings.



WORKING / DECISION

Two measured signal strengths are given. Construct the percentage model and the requested forecast.

8. Water volume follows $V = 480 - 16t$, where V is litres and t is minutes after a valve opens. Interpret both numbers, find when the tank empties and state a sensible domain. Draw the physically meaningful part of the graph.



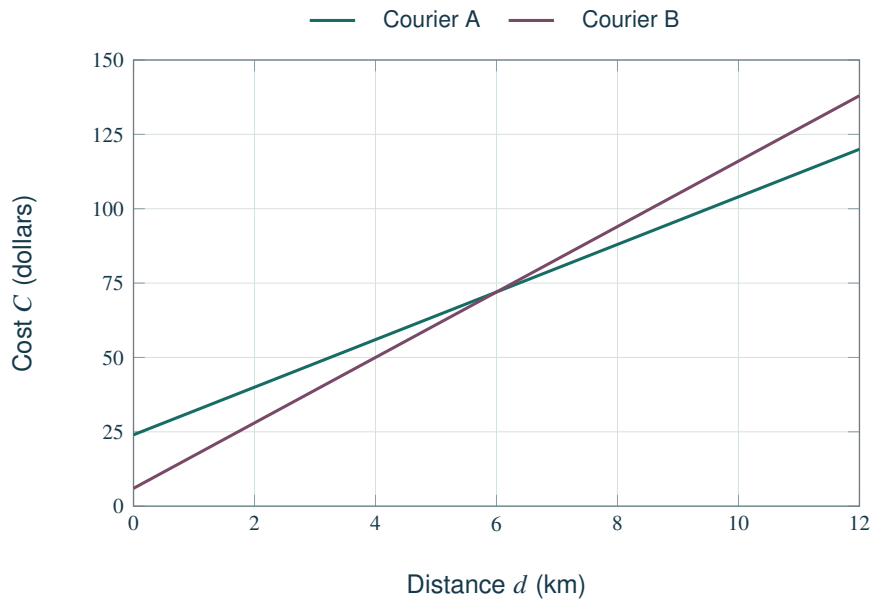
WORKING / DECISION

The initial tank volume is supplied. Draw the model only while its physical interpretation is valid.

3. Make the decision

Use the graph to record your thinking. Show calculations and include units where needed.

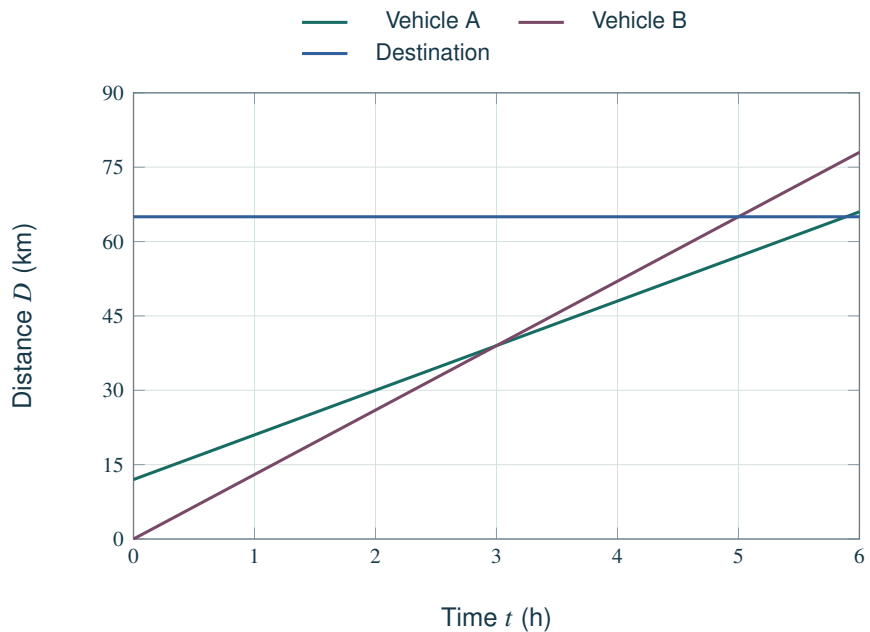
9. Courier A charges $C = 8d + 24$ and Courier B charges $C = 11d + 6$, in dollars for d kilometres. Find the distance at which the costs match, and choose the cheaper courier for 10 km. Mark the comparison on the graph.



WORKING / DECISION

The two supplied price rules are plotted for comparison. Calculate and mark the matching point.

10. Rescue vehicles travel on the same road. A is at $D = 9t + 12$ km and B at $D = 13t$ km after t hours. Find when and where B catches A. Which reaches the 65 km point first? Use the common graph to support your decision.



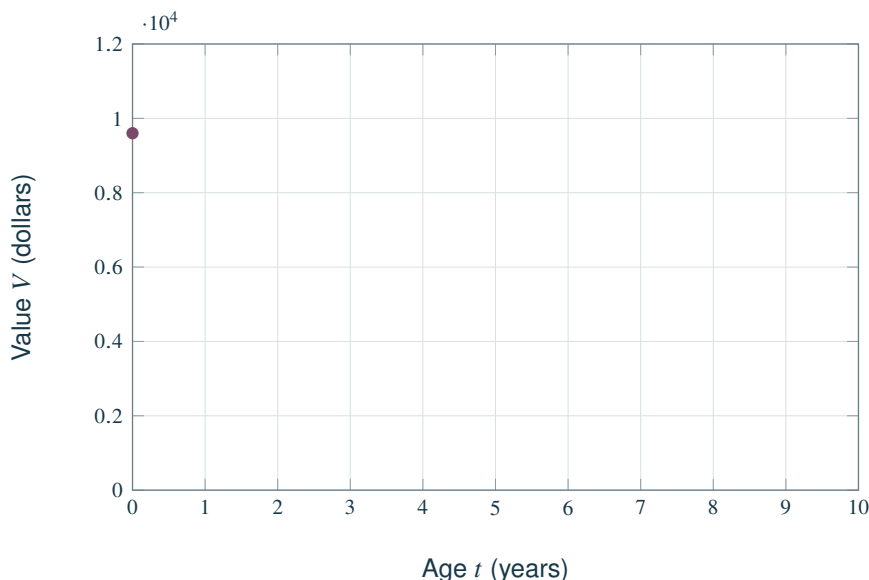
WORKING / DECISION

The supplied distance models and common destination are drawn on one set of axes.

3. Make the decision

Use the graph to record your thinking. Show calculations and include units where needed.

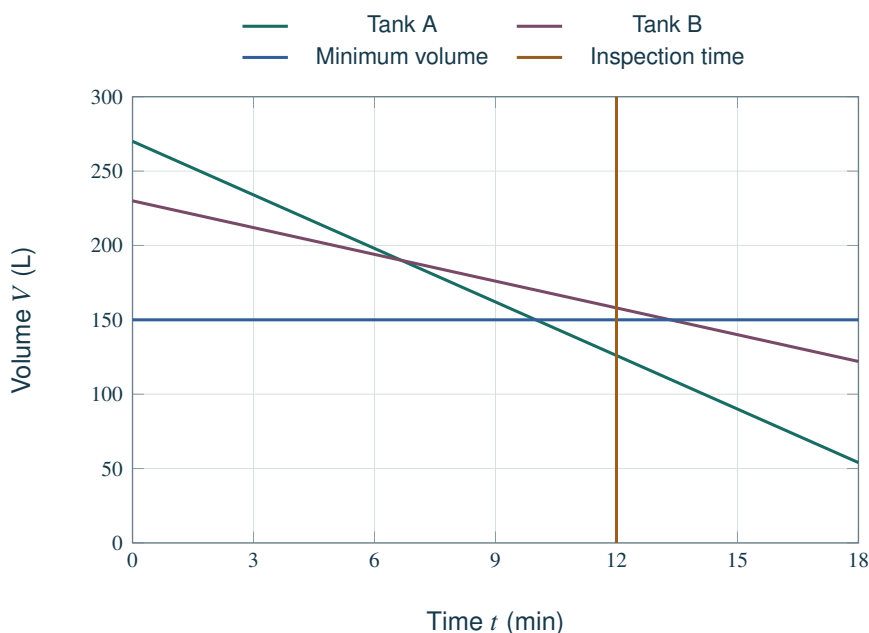
- 11.** A machine costs \$9,600 and loses \$1,200 of value each year under a straight-line model. Write $V(t)$, find its value after 5 years and explain why the model should not continue unchanged past 8 years. Show the valid model on the graph.



WORKING / DECISION

Only the original machine value is given. Construct the depreciation model over a sensible domain.

- 12.** A water station must hold at least 150 L after 12 minutes. Tank A follows $V = 270 - 12t$ and Tank B follows $V = 230 - 6t$. Choose a suitable tank, state its remaining volume and find how many minutes it can operate before reaching 150 L. Mark your decision on the comparison graph.



WORKING / DECISION

The two given tank models, inspection time and minimum operating volume are shown.

6I / Worked answers

Teacher appendix. Equivalent exact forms are accepted. Use the reasoning to check student work.

1. $y = 2x + 4$.

- $m = \frac{10-2}{3-(-1)} = \frac{8}{4} = 2$.
 - $2 = 2(-1) + c$ gives $c = 4$. Checks: 2 and 10.
-

2. $H = -\frac{3}{4}d + \frac{25}{2}$.

- $m = \frac{5-11}{10-2} = -\frac{6}{8} = -\frac{3}{4}$.
 - $11 = -\frac{3}{4}(2) + c$, so $c = 12.5$.
-

3. Yes; $D = 7t + 14$.

- The changes are 21 MB in each 3-minute interval, a constant 7 MB/min.
 - At $t = 0$ the stored data is 14 MB.
-

4. No; the rates are 3 and 4.5 degrees per minute.

- First rate $= \frac{24-18}{2} = 3$.
 - Second rate $= \frac{33-24}{2} = 4.5$. Unequal rates rule out one exact line.
-

5. $M = 1.75n + 12$; 26 kg for 8 panels.

- 1.75 kg/panel is the added mass; 12 kg is the empty frame.
 - $M = 1.75(8) + 12 = 26$.
-

6. $C = 28h + 65$; 7 hours.

- Set $261 = 28h + 65$.
 - $196 = 28h$ gives $h = 7$.
-

6I / Worked answers

Teacher appendix. Equivalent exact forms are accepted. Use the reasoning to check student work.

7. $S = -4d + 100$; 40% at 15 km.

- $m = \frac{60-84}{10-4} = -4$ percentage points/km; $c = 100$.
 - $S(15) = -60 + 100 = 40$. A proportion model would instead be $S = -0.04d + 1$.
-

8. Starts with 480 L; loses 16 L/min; empty at 30 min; $0 \leq t \leq 30$.

- Set $V = 0$: $16t = 480$, so $t = 30$.
 - Beyond 30 minutes the line predicts negative volume, which is not physical.
-

9. Match at 6 km (\$72); Courier A is cheaper at 10 km (\$104 versus \$116).

- $8d + 24 = 11d + 6$ gives $18 = 3d$.
 - Evaluate both at $d = 10$.
-

10. Meet at 3 h, 39 km. B reaches 65 km first, at 5 h.

- $9t + 12 = 13t$ gives $t = 3$ and $D = 39$.
 - A needs $\frac{65-12}{9} = \frac{53}{9}$ h; B needs $\frac{65}{13} = 5$ h.
-

11. $V = 9600 - 1200t$; \$3,600 after 5 years; value reaches zero at 8 years.

- $V(5) = 9600 - 6000 = 3600$.
 - At $t > 8$ the rule gives negative value, so the physical interpretation fails.
-

12. Tank B; 158 L after 12 minutes; reaches 150 L at $\frac{40}{3}$ minutes.

- $A(12) = 126 < 150$, while $B(12) = 158 \geq 150$.
 - $150 = 230 - 6t$ gives $t = \frac{80}{6} = \frac{40}{3}$ minutes (13 min 20 s).
-

6J / CLASSROOM LESSON

Direct variation

Name: _____ Class: _____ Date: _____

Today you will

- Build and use a direct-variation model.
- Recognise a proportional relationship from its graph.

Keep these ideas

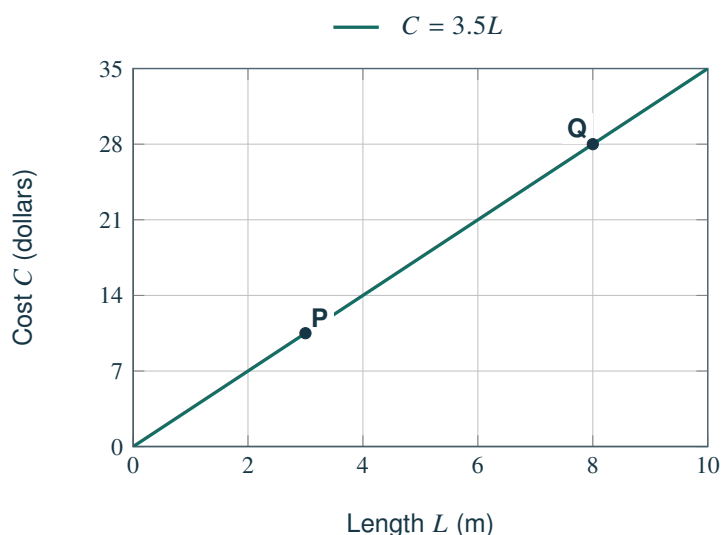
Direct variation has the form $y = kx$; $k = \frac{y}{x}$ for $x \neq 0$.

Its graph is a straight line through $(0, 0)$.

A fixed non-zero charge means the total is not directly proportional to the quantity.

Worked example

Ribbon costs \$10.50 for 3 m, with no fixed charge. Find a cost model and the length that costs \$28.



1. $k = \frac{10.50}{3} = 3.50$ dollars per metre, so $C = 3.5L$.

2. $28 = 3.5L$, so $L = \frac{28}{3.5} = 8$ m.

Answer

$C = 3.5L$; \$28 buys 8 m.

The worked model links ribbon length and cost.

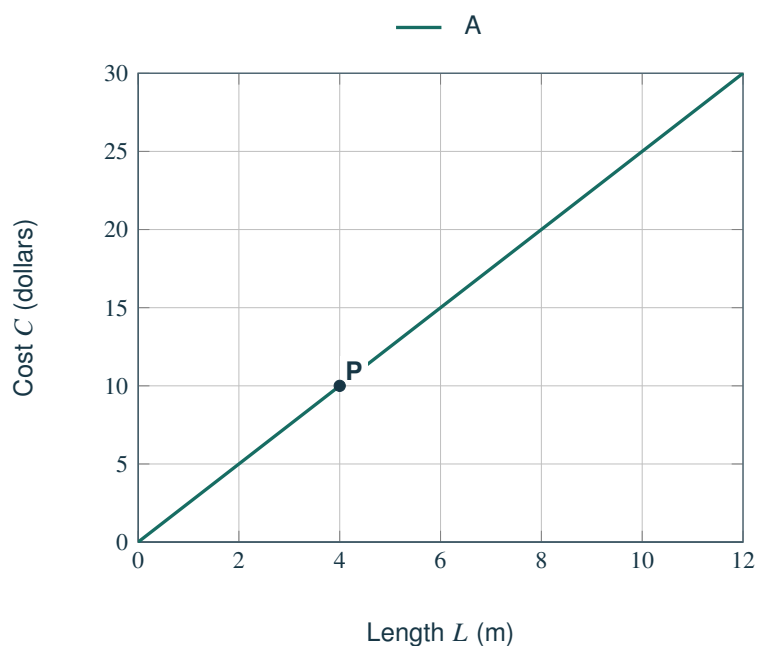
Your turn

1. M varies directly with V . If $M = 9.9$ when $V = 4.5$, find k and the model.

2. A liquid has mass $M = 2.2V$, in kilograms. Find its volume V , in litres, when $M = 33$.

Apply it on the graph

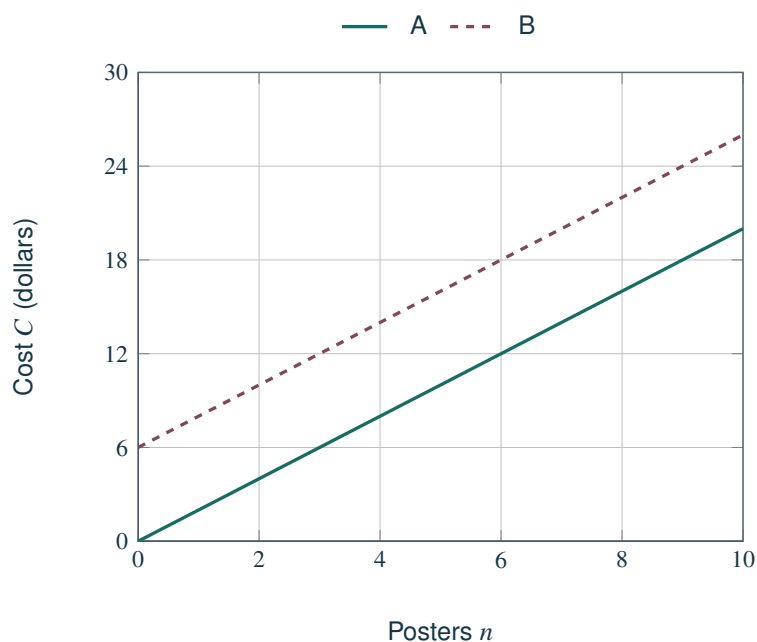
Write your calculations beside each diagram. Label any points or curves you add.



Questions 3-4: Line A and the given point $P(4, 10)$.

3. Line A shows cord cost. Use the marked point $P(4, 10)$ to find the price per metre and write a cost model.

4. Use the same graph as Question 3. How many metres of cord can you buy for \$30? Confirm your reading by calculation.



Questions 5-6: The supplied price lines for services A and B.

5. The graph compares two printing services. Which service has cost directly proportional to the number of posters? Explain using the graph.

6. Use the same graph as Question 5. You need 9 posters and have \$20. Calculate both prices and choose an affordable service.

6J / HOMEWORK

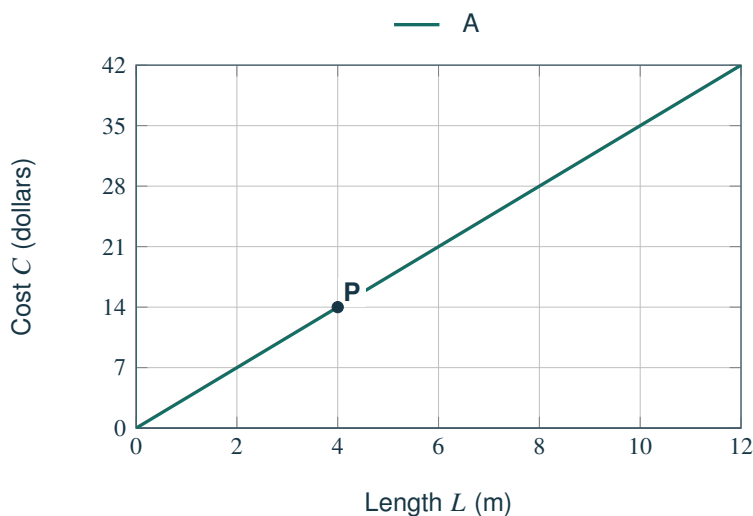
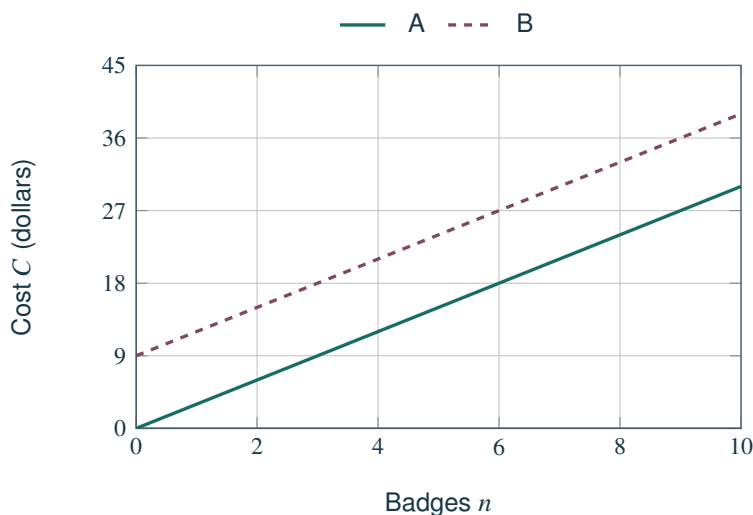
Direct variation

Name: _____ Class: _____ Date: _____

Use your classroom lesson if needed. Show your working; label all graphs.

1. M varies directly with V . If $M = 18$ when $V = 7.5$, find k and the model.

2. A fluid has mass $M = 2.4V$, in kilograms. Find its volume V , in litres, when $M = 42$.

Questions 3-4: Line A and the given point $P(4, 14)$.

Questions 5-6: The supplied price lines for suppliers A and B.

3. Line A shows cable cost. Use $P(4, 14)$ to find the price per metre and write a cost model.

4. Use the same graph as Question 3. How many metres of cable can you buy for \$35? Confirm by calculation.

5. The graph compares two badge suppliers. Which supplier's cost is directly proportional to the number of badges? Explain using the graph.

6. Use the same graph as Question 5. You need 7 badges and have \$24. Calculate both prices and choose an affordable supplier.

6J / Classroom answers

Teacher copy. Accept equivalent exact forms and valid alternative reasoning.

1. Find a direct model

$k = 2.2$; $M = 2.2V$.

- $k = \frac{9.9}{4.5} = 2.2$.
-

2. Reverse a direct model

15 L.

- $33 = 2.2V$ gives $V = \frac{33}{2.2} = 15$.
-

3. Read a proportional graph

\$2.50 per metre; $C = 2.5L$.

- $k = \frac{10}{4} = 2.5$; the line passes through the origin.
-

4. Use a graph model

12 m.

- $30 = 2.5L$ gives $L = 12$; this agrees with the graph.
-

5. Distinguish a fixed charge

Service A; its straight line passes through the origin. B has a \$6 fixed charge.

- A has $C = 2n$; B has $C = 2n + 6$.
-

6. Choose a feasible plan

A costs \$18 and B costs \$24. Only A fits the budget.

- $C_A = 2(9) = 18$; $C_B = 2(9) + 6 = 24$. Compare each total with \$20.
-

6J / Homework answers

Teacher copy. Accept equivalent exact forms and valid alternative reasoning.

1. Find a direct model

$k = 2.4$; $M = 2.4V$.

- $k = \frac{18}{7.5} = 2.4$.
-

2. Reverse a direct model

17.5 L.

- $42 = 2.4V$ gives $V = \frac{42}{2.4} = 17.5$.
-

3. Read a proportional graph

\$3.50 per metre; $C = 3.5L$.

- $k = \frac{14}{4} = 3.5$; the line passes through the origin.
-

4. Use a graph model

10 m.

- $35 = 3.5L$ gives $L = 10$.
-

5. Distinguish a fixed charge

A; its straight line passes through the origin. B has a \$9 fixed charge.

- A has $C = 3n$; B has $C = 3n + 9$.
-

6. Choose a feasible plan

A costs \$21 and B costs \$30. Only A fits the budget.

- $C_A = 3(7) = 21$; $C_B = 3(7) + 9 = 30$. Compare each total with \$24.
-

6K / CLASSROOM LESSON

Inverse variation

Name: _____ Class: _____ Date: _____

Today you will

- Build and use a model with a constant product.
- Read inverse graphs and explain their branches and excluded values.

Keep these ideas

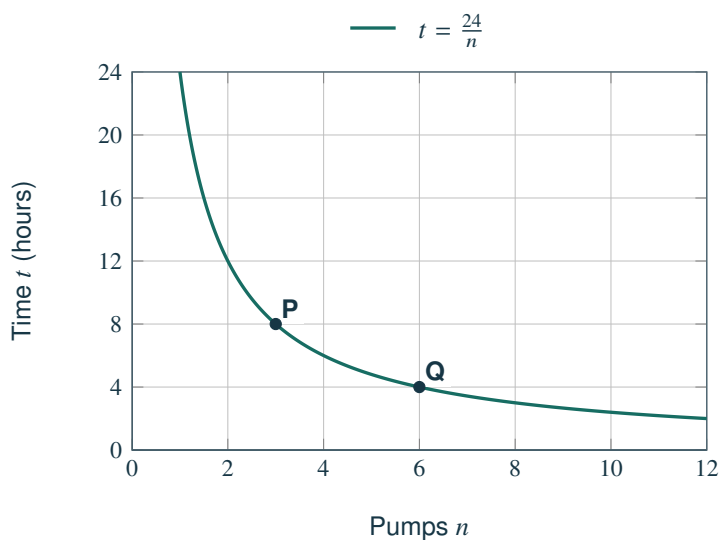
Inverse variation has $y = \frac{k}{x}$, so the product $xy = k$ stays constant.

For non-zero k , the graph is a hyperbola; it never reaches either axis and $x \neq 0$.

Multiplying x by a positive factor divides y by that factor. A decreasing straight line is not an inverse graph.

Worked example

Three identical pumps empty a tank in 8 hours. The time is inversely proportional to pump count. Find the model and the time for six pumps.



1. $k = nt = 3 \times 8 = 24$ pump-hours, so $t = \frac{24}{n}$.

2. With six pumps, $t = \frac{24}{6} = 4$ hours.

Answer

$t = \frac{24}{n}$; six pumps take 4 hours.

The worked model for positive pump counts.

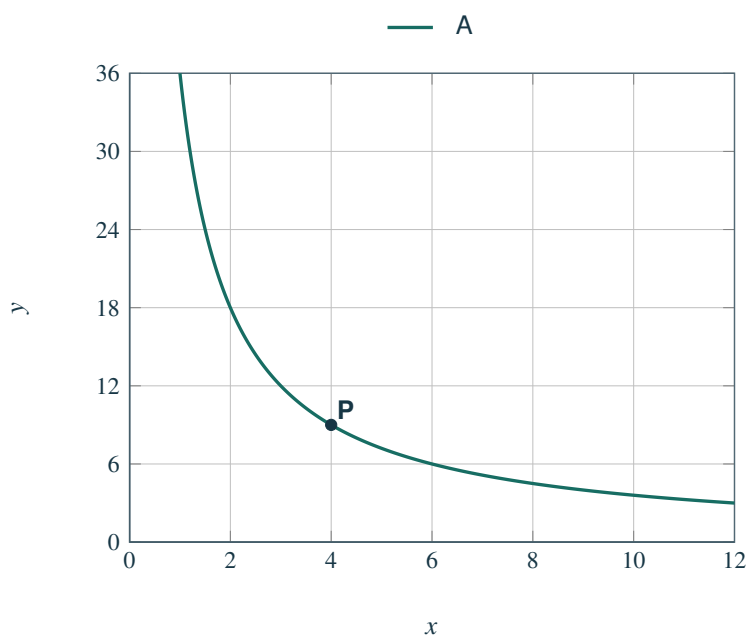
Your turn

1. y varies inversely with x . If $y = 15$ when $x = 4$, find k and the model.

2. For $y = \frac{60}{x}$, find x when $y = 8$.

Apply it on the graph

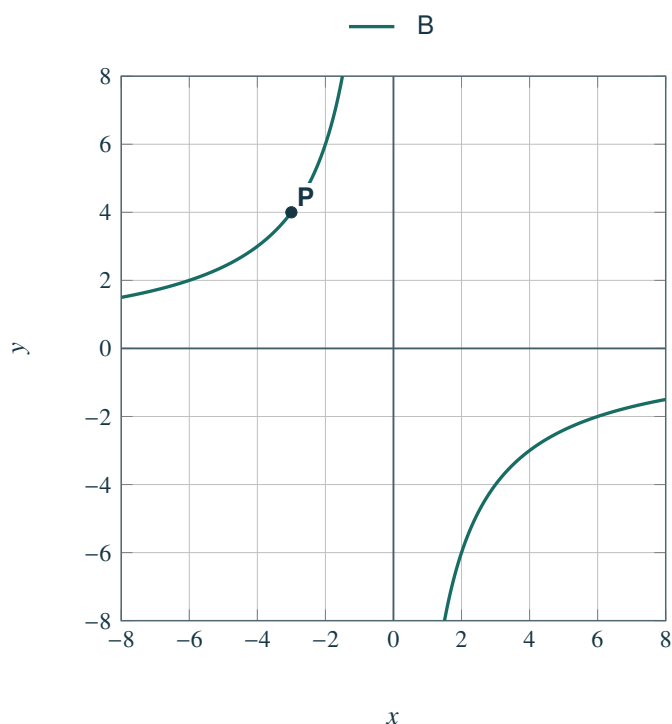
Write your calculations beside each diagram. Label any points or curves you add.



Questions 3-4: Positive branch A, including the given point $P(4, 9)$.

3. Graph A shows inverse variation. Use the given point $P(4, 9)$ to find k and write its equation.

4. Use the same graph as Question 3. Find y at $x = 9$. Describe what happens to y when x triples from 3 to 9.



Questions 5-6: Both branches of inverse graph B; $P(-3, 4)$ is given.

5. Graph B shows inverse variation. Use $P(-3, 4)$ to find k and its equation. State which quadrants contain its two branches.

6. Use the same graph as Question 5. Does B cross either axis? Explain why, and state the excluded value of x .

6K / HOMEWORK

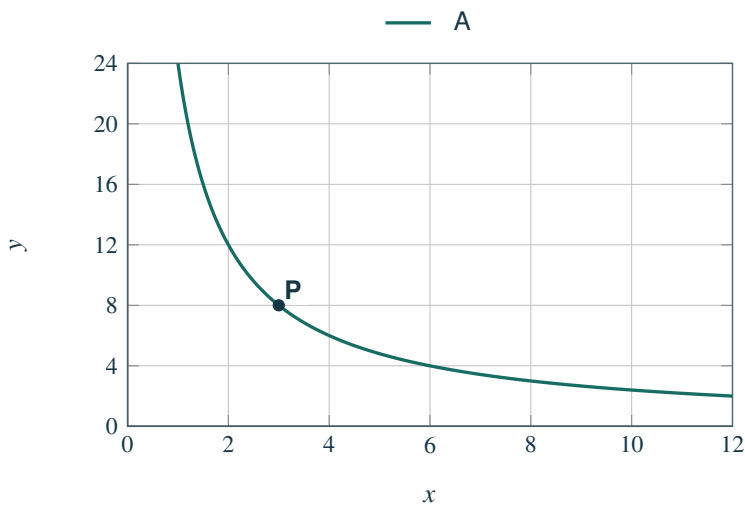
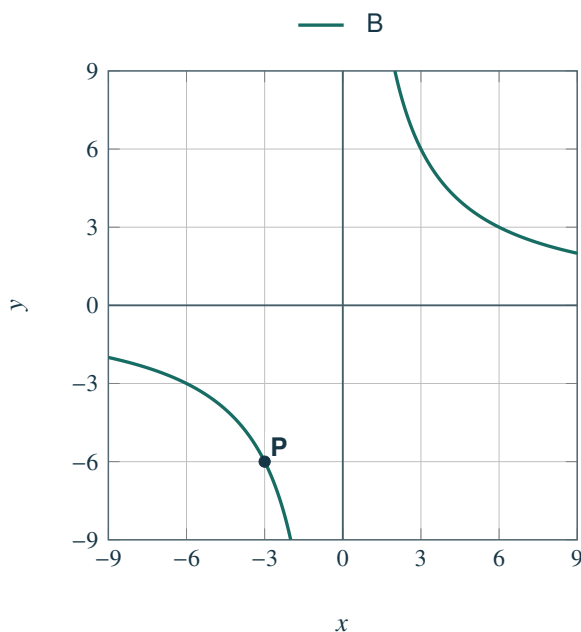
Inverse variation

Name: _____ Class: _____ Date: _____

Use your classroom lesson if needed. Show your working; label all graphs.

1. y varies inversely with x . If $y = 14$ when $x = 6$, find k and the model.

2. For $y = \frac{84}{x}$, find x when $y = 12$.

Questions 3-4: Positive branch A, including the given point $P(3, 8)$.Questions 5-6: Both branches of inverse graph B; $P(-3, -6)$ is given.

3. Graph A shows inverse variation. Use $P(3, 8)$ to find k and write its equation.

4. Use the same graph as Question 3. Find y at $x = 8$. Describe what happens to y when x increases from 2 to 8.

5. Graph B shows inverse variation. Use $P(-3, -6)$ to find k and its equation. State which quadrants contain its branches.

6. Use the same graph as Question 5. Can a point with $x = 0$ or $y = 0$ lie on B? Explain.

6K / Classroom answers

Teacher copy. Accept equivalent exact forms and valid alternative reasoning.

1. Find an inverse model

$$k = 60; y = \frac{60}{x}.$$

- $k = xy = 4 \times 15 = 60.$
-

2. Reverse an inverse model

$$x = 7.5.$$

- $8x = 60$ gives $x = \frac{60}{8} = 7.5.$
-

3. Read an inverse graph

$$k = 36; y = \frac{36}{x}.$$

- $k = xy = 4 \times 9 = 36.$
-

4. Scale an inverse relationship

$y = 4$. The value of y is divided by 3, from 12 to 4.

- At $x = 9$, $y = \frac{36}{9} = 4$. At $x = 3$, $y = 12$.
-

5. Interpret signed branches

$k = -12$; $y = -\frac{12}{x}$. The branches are in quadrants II and IV.

- $k = (-3)(4) = -12$. The coordinates have opposite signs.
-

6. Explain axes and domain

Neither axis is crossed; $x = 0$ is excluded.

- Every point satisfies $xy = -12$. On either axis, $xy = 0$, so it cannot lie on B. Division by zero excludes $x = 0$.
-

6K / Homework answers

Teacher copy. Accept equivalent exact forms and valid alternative reasoning.

1. Find an inverse model

$$k = 84; y = \frac{84}{x}.$$

- $k = xy = 6 \times 14 = 84.$
-

2. Reverse an inverse model

$$x = 7.$$

- $12x = 84$ gives $x = 7.$
-

3. Read an inverse graph

$$k = 24; y = \frac{24}{x}.$$

- $k = xy = 3 \times 8 = 24.$
-

4. Scale an inverse relationship

$y = 3$. Multiplying x by 4 divides y by 4, from 12 to 3.

- At $x = 8$, $y = \frac{24}{8} = 3$. At $x = 2$, $y = 12.$
-

5. Interpret signed branches

$k = 18$; $y = \frac{18}{x}$. The branches are in quadrants I and III.

- $k = (-3)(-6) = 18$. The coordinates have the same sign.
-

6. Explain axes and domain

No. Neither axis contains a point of B, and $x = 0$ is excluded.

- The model has $xy = 18$. If either coordinate were zero, the product would be zero instead.
-

6L / CLASSROOM LESSON

Exploring parabolas

Name: _____ Class: _____ Date: _____

Today you will

- Read a parabola's vertex, symmetry line and intercepts.
- Use a quadratic model to check the shape and clearance of a structure.

Keep these ideas

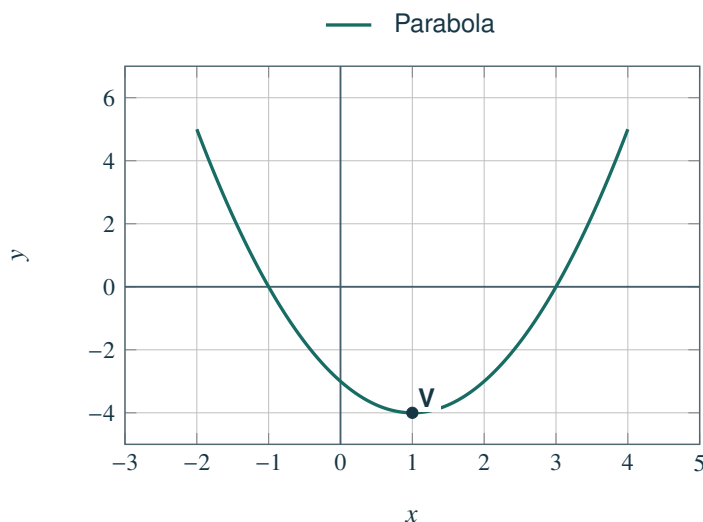
In $y = a(x - h)^2 + k$, the vertex is (h, k) and the symmetry line is $x = h$.

A positive a opens upward; a negative a opens downward. A larger $|a|$ makes the curve narrower on the same axes.

At an x -intercept, $y = 0$. Points equally far from the symmetry line have the same height.

Worked example

For $y = (x - 1)^2 - 4$, locate the vertex and find both x -intercepts.



1. At $x = 1$, the squared part is zero, so $y = -4$. The vertex is $(1, -4)$.
2. For the intercepts, $0 = (x - 1)^2 - 4$, so $(x - 1)^2 = 4$. Thus $x - 1 = 2$ or $x - 1 = -2$.

Answer

Vertex $(1, -4)$; x -intercepts $(-1, 0)$ and $(3, 0)$.

The curve shows the worked-example model; V marks its lowest point.

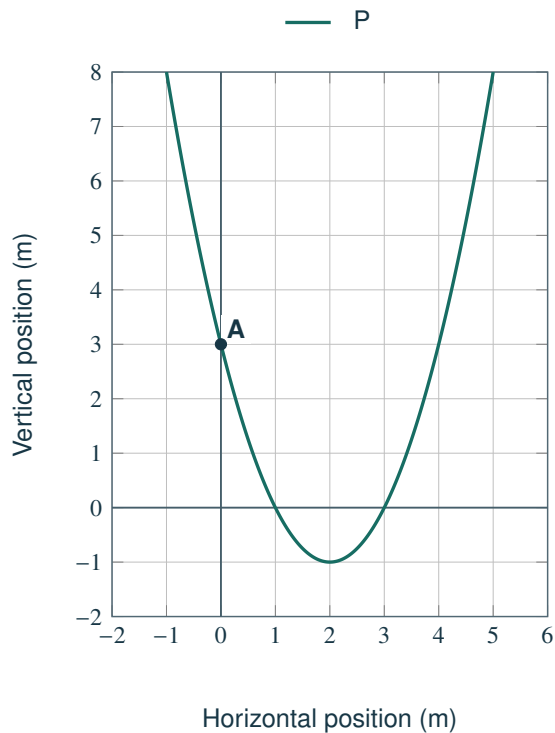
Your turn

1. For $y = -2x^2 + 8$, find y when $x = -2$.

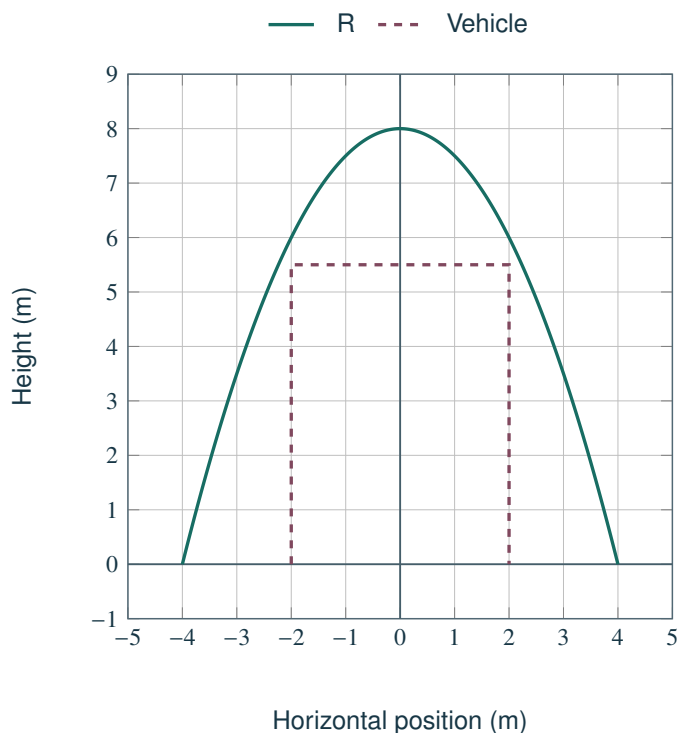
2. Compared with $y = x^2$, describe the opening direction and width of $y = \frac{1}{4}x^2$.

Apply it on the graph

Write your calculations beside each diagram. Label any points or curves you add.



Frame P is measured on site coordinates. Sensor A is at (0, 3) m.



Arch R and the vehicle cross-section for Questions 5 and 6. Ground level is $y = 0$.

3. Read frame P's vertex from the graph. Write the equation of its symmetry line.

4. On the same graph P, sensor A is at (0, 3). Give its matching point across the symmetry line and the distance between the sensors.

5. Arch R follows $y = -\frac{1}{2}x^2 + 8$, in metres. Calculate its two ground crossings and total width.

6. On the same graph R, the vehicle spans $x = -2$ to $x = 2$ and is 5.5 m high. Does it fit with 0.5 m roof clearance? Justify.

6L / HOMEWORK

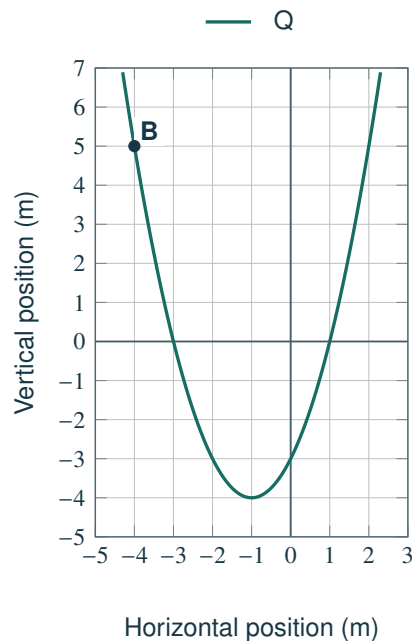
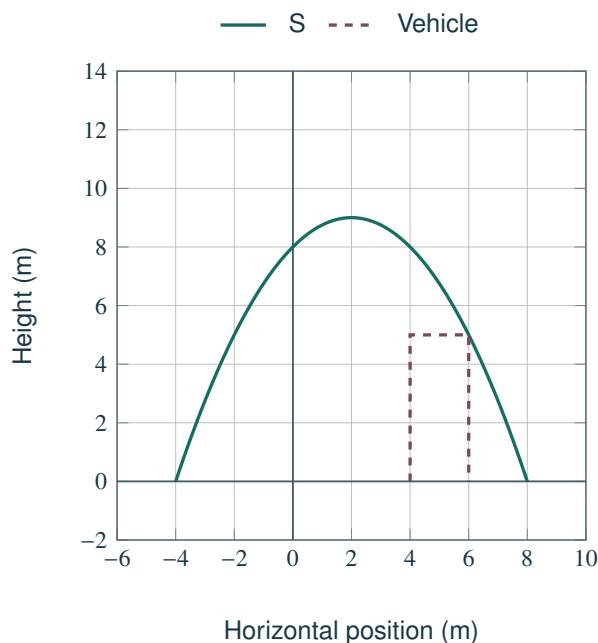
Exploring parabolas

Name: _____ Class: _____ Date: _____

Use your classroom lesson if needed. Show your working; label all graphs.

1. For $y = -3x^2 + 12$, find y when $x = -1$.

2. Compared with $y = x^2$, describe the opening direction and width of $y = -2x^2$.

Frame Q is measured on site coordinates. Sensor B is at $(-4, 5)$ m.Arch S and the vehicle cross-section for Questions 5 and 6. Ground level is $y = 0$.

3. Read frame Q's vertex from the graph. Write the equation of its symmetry line.

4. On the same graph Q, sensor B is at $(-4, 5)$. Give its matching point across the symmetry line and the distance between the sensors.5. Arch S follows $y = -\frac{1}{4}(x - 2)^2 + 9$, in metres. Calculate its two ground crossings and total width.6. On the same graph S, the vehicle spans $x = 4$ to $x = 6$ and is 5 m high. Does it fit with 0.5 m roof clearance? Justify.

6L / Classroom answers

Teacher copy. Accept equivalent exact forms and valid alternative reasoning.

1. Evaluate a quadratic

$$y = 0.$$

- $y = -2(-2)^2 + 8 = -2(4) + 8 = 0.$

2. Opening and width

It opens upward and is wider.

- The coefficient is positive. Its magnitude, $\frac{1}{4}$, is less than 1, making a wider curve on the same axes.

3. Vertex and symmetry

Vertex $(2, -1)$; symmetry line $x = 2$.

- The lowest point is $(2, -1)$, and the matching halves lie on either side of the vertical line $x = 2$.

4. Reflected points

Matching point $(4, 3)$; distance 4 m.

- A is 2 m left of $x = 2$. Its matching point is 2 m right, at $(4, 3)$.
- The horizontal separation is $4 - 0 = 4$ m.

5. Intercepts and span

Crossings $(-4, 0)$ and $(4, 0)$; width 8 m.

- At ground level, $0 = -\frac{1}{2}x^2 + 8$, so $x^2 = 16$ and $x = \pm 4$.
- The width is $4 - (-4) = 8$ m.

6. Clearance with parabolas

Yes. The smallest gap above its roof is exactly 0.5 m.

- The arch is lowest over the vehicle at its outer edges: $y = -\frac{1}{2}(2)^2 + 8 = 6$ m.
- The available gap is $6 - 5.5 = 0.5$ m, so the whole roof meets the requirement.

6L / Homework answers

Teacher copy. Accept equivalent exact forms and valid alternative reasoning.

1. Evaluate a quadratic

$$y = 9.$$

- $y = -3(-1)^2 + 12 = -3 + 12 = 9.$

2. Opening and width

It opens downward and is narrower.

- The coefficient is negative, so the curve is reflected downward. Its magnitude is $2 > 1$, making it narrower on the same axes.
-

3. Vertex and symmetry

Vertex $(-1, -4)$; symmetry line $x = -1$.

- The lowest point is $(-1, -4)$, so the matching halves are separated by the vertical line $x = -1$.
-

4. Reflected points

Matching point $(2, 5)$; distance 6 m.

- B is 3 m left of $x = -1$. Its partner is 3 m right, at $(2, 5)$.
 - The separation is $2 - (-4) = 6$ m.
-

5. Intercepts and span

Crossings $(-4, 0)$ and $(8, 0)$; width 12 m.

- $0 = -\frac{1}{4}(x - 2)^2 + 9$ gives $(x - 2)^2 = 36$, so $x - 2 = \pm 6$.
 - Thus $x = -4$ or $x = 8$, and the width is $8 - (-4) = 12$ m.
-

6. Clearance with parabolas

No. At the right edge, the roof touches the arch, leaving no safety gap.

- The arch is lowest over this lane at $x = 6$: $y = -\frac{1}{4}(6 - 2)^2 + 9 = 5$ m.
 - The gap is $5 - 5 = 0$ m, less than the required 0.5 m.
-

6M / CLASSROOM LESSON

Exponential graphs

Name: _____ Class: _____ Date: _____

Today you will

- Evaluate exponential rules, including negative inputs.
- Compare growth, decay and practical whole-step choices.

Keep these ideas

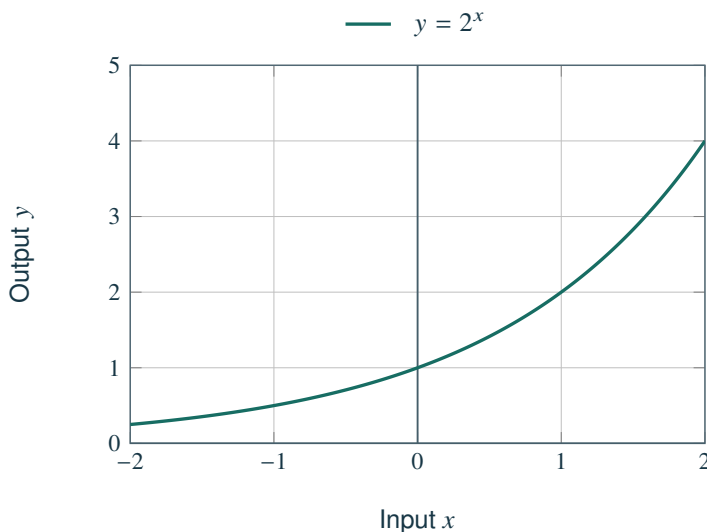
For $y = a^x$, use $a > 0$ and $a \neq 1$. Every such graph passes through $(0, 1)$.

A negative exponent means a reciprocal: $a^{-n} = \frac{1}{a^n}$.

If $a > 1$ the graph grows; if $0 < a < 1$ it decays. Outputs stay positive for every finite input.

Worked example

For $y = 2^x$, find the outputs at $x = -2, 0, 2$ and state the vertical-axis intercept.



1. $2^{-2} = \frac{1}{2^2} = \frac{1}{4}$; $2^0 = 1$; $2^2 = 4$.

2. The vertical axis has input $x = 0$, so the intercept is $(0, 1)$.

Answer

Outputs $\frac{1}{4}, 1, 4$; vertical-axis intercept $(0, 1)$.

The curve used in the worked example.

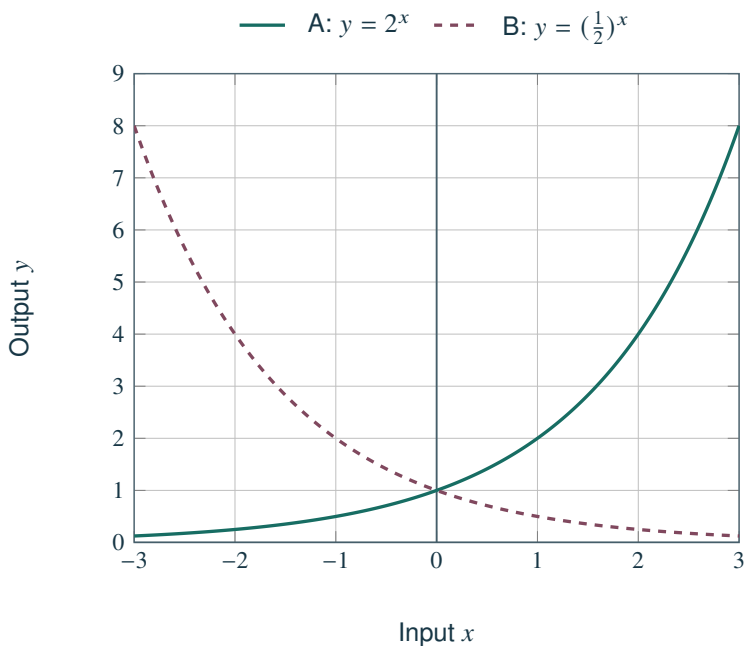
Your turn

1. Evaluate 3^{-2} and 3^0 .

2. Does $y = 0.4^x$ grow or decay? State its vertical-axis intercept.

Apply it on the graph

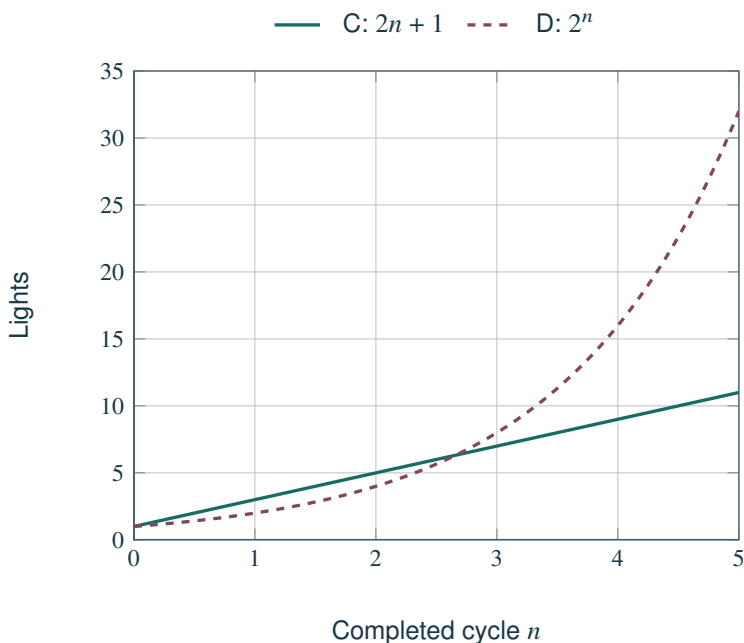
Write your calculations beside each diagram. Label any points or curves you add.



Both curves are supplied. Use this graph for Questions 3 and 4.

3. For curves A and B shown, find both outputs at $x = 2$. State the reflection linking their graphs.

4. Using the same graph, give the shared vertical-axis intercept. Does either curve cross the horizontal axis? Explain.



Plan C adds two lights per cycle; Plan D doubles. Read whole cycles only.

5. For the plans shown, find the first whole cycle with more lights under Plan D than Plan C.

6. Using the same graph, choose the smallest controller for Plan D at cycle 5: 30, 36 or 48 lights. State the final count.

6M / HOMEWORK

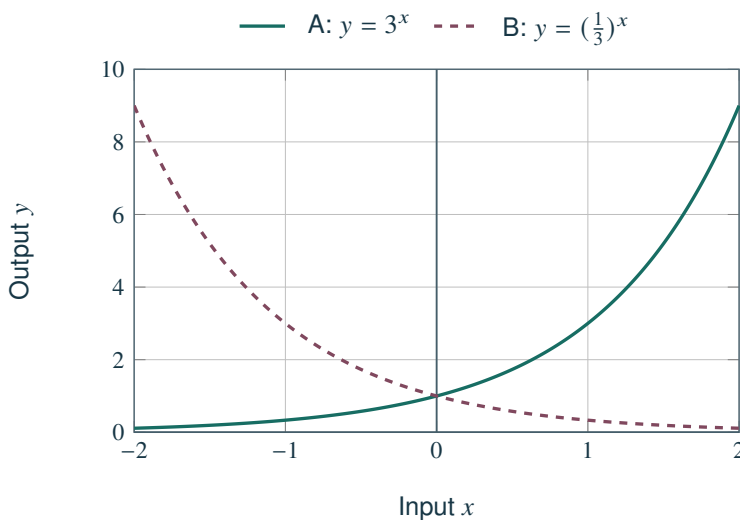
Exponential graphs

Name: _____ Class: _____ Date: _____

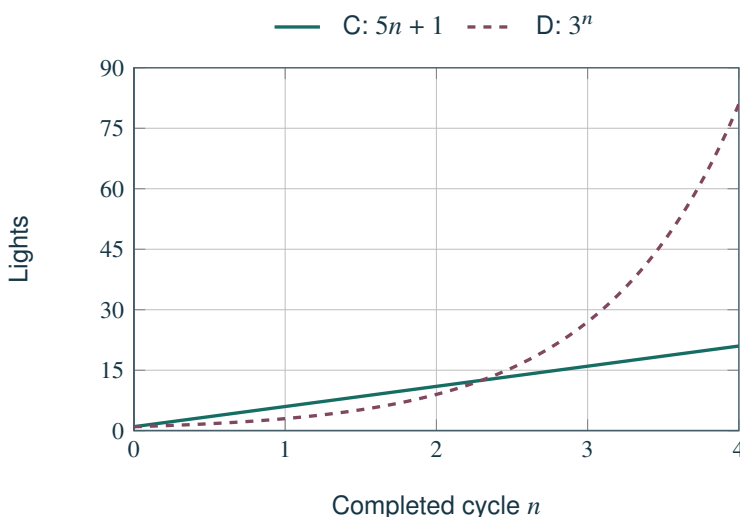
Use your classroom lesson if needed. Show your working; label all graphs.

1. Evaluate 4^{-2} and 4^0 .

2. Does $y = 0.25^x$ grow or decay? State its vertical-axis intercept.



The two rules and their curves are given. Use this graph for Questions 3 and 4.



Plan C adds five lights per cycle; Plan D triples. Read whole cycles only.

3. For curves A and B shown, find both outputs at $x = 1$. State the reflection linking their graphs.

4. Using the same graph, give the shared vertical-axis intercept. Can curve B become exactly zero at a finite input? Explain.

5. For the plans shown, find the first whole cycle with more lights under Plan D than Plan C.

6. Using the same graph, choose the smallest controller for Plan D at cycle 4: 70, 90 or 110 lights. State the final count.

6M / Classroom answers

Teacher copy. Accept equivalent exact forms and valid alternative reasoning.

1. Negative and zero exponents

$\frac{1}{9}$ and 1.

- $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$; any positive base to power zero gives 1.
-

2. Growth, decay and intercepts

Decay; (0, 1).

- The base lies between 0 and 1; $0.4^0 = 1$.
-

3. Reciprocal-base reflection

A: 4; B: $\frac{1}{4}$. Reflection across the vertical axis.

- $2^2 = 4$ and $(\frac{1}{2})^2 = \frac{1}{4}$.
 - Replacing x by $-x$ gives $(\frac{1}{2})^x = 2^{-x}$.
-

4. Positive outputs

Shared intercept (0, 1); neither crosses the horizontal axis.

- At $x = 0$, both outputs are 1.
 - Every finite input produces a positive output, not zero.
-

5. Exponential versus linear

Cycle 3.

- At cycle 2, C has 5 and D has 4.
 - At cycle 3, C has 7 and D has 8; earlier whole cycles do not give D a lead.
-

6. Choose a capacity

32 lights; choose the 36-light controller.

- $D(5) = 2^5 = 32$.
 - A capacity of 30 is too small; 36 is the smallest available capacity at least 32.
-

6M / Homework answers

Teacher copy. Accept equivalent exact forms and valid alternative reasoning.

1. Negative and zero exponents

$\frac{1}{16}$ and 1.

- $4^{-2} = \frac{1}{4^2} = \frac{1}{16}$ and $4^0 = 1$.
-

2. Growth, decay and intercepts

Decay; (0, 1).

- The base lies between 0 and 1; $0.25^0 = 1$.
-

3. Reciprocal-base reflection

A: 3; B: $\frac{1}{3}$. Reflection across the vertical axis.

- $3^1 = 3$ and $(\frac{1}{3})^1 = \frac{1}{3}$.
 - The reciprocal-base curve equals 3^{-x} .
-

4. Positive outputs

(0, 1); no finite input gives zero.

- Both positive bases raised to zero give 1.
 - Repeated multiplication by a positive fraction stays positive.
-

5. Exponential versus linear

Cycle 3.

- At cycle 2, C has 11 and D has 9.
 - At cycle 3, C has 16 and D has 27; earlier whole cycles do not give D a lead.
-

6. Choose a capacity

81 lights; choose the 90-light controller.

- $D(4) = 3^4 = 81$.
 - A capacity of 70 is too small; 90 is the smallest suitable capacity.
-

6N / CLASSROOM LESSON

Exponential growth and decay

Name: _____ Class: _____ Date: _____

Today you will

- Build models for repeated percentage changes.
- Use whole-period comparisons to make justified decisions.

Keep these ideas

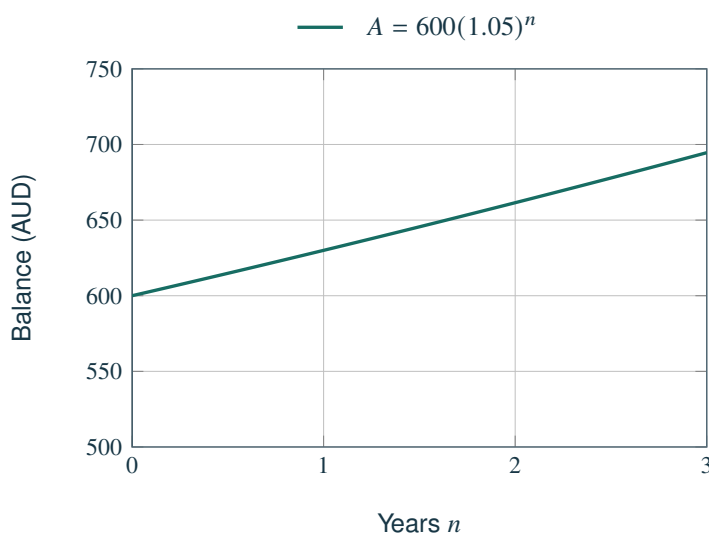
An increase of $r\%$ uses multiplier $q = 1 + \frac{r}{100}$; a decrease uses $q = 1 - \frac{r}{100}$.

After n periods, $A = A_0q^n$, where A_0 is the initial amount.

Keep the full multiplier and intermediate values. Round the final money amount to cents; check the previous whole period for an earliest-time decision.

Worked example

A fund starts at \$600 and grows by 5% each year. Find its balance after two years.



1. The multiplier is $q = 1.05$, so $A = 600(1.05)^n$.

2. At $n = 2$, $A = 600(1.05)^2 = 661.5$.

Answer
\$661.50.

The savings model used in the worked example. Annual balances occur at whole years.

Your turn

1. Write the multipliers for a 9% increase and a 14% decrease.

2. A \$1,200 camera loses 15% of its remaining value yearly. Write its model and value after two years.

Apply it on the graph

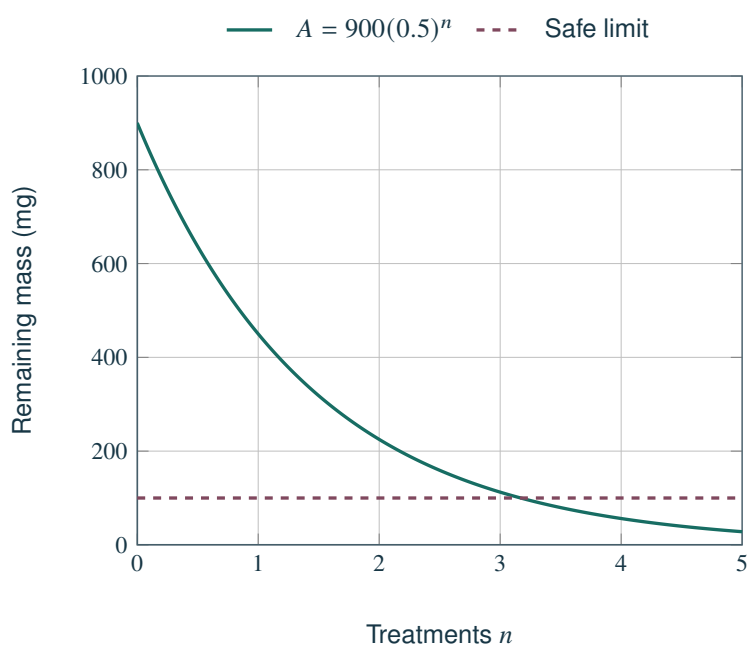
Write your calculations beside each diagram. Label any points or curves you add.



Savings begin at \$800; the purchase price is \$1,050. Use this graph for Questions 3 and 4.

3. The graph shows $S = 800(1.08)^n$. Find the savings after three years, rounded to cents.

4. Using the same graph, find the first whole year when savings cover \$1,050. Give the balance in that year.



Contaminant amount and the 100 mg limit are supplied. Read whole treatments only.

5. The sample follows $A = 900(0.5)^n$ mg. How much remains after three treatments? Is it at or below the 100 mg limit?

6. Using the same graph, find the first safe whole treatment cycle. State the amount at that cycle and one cycle earlier.

6N / HOMEWORK

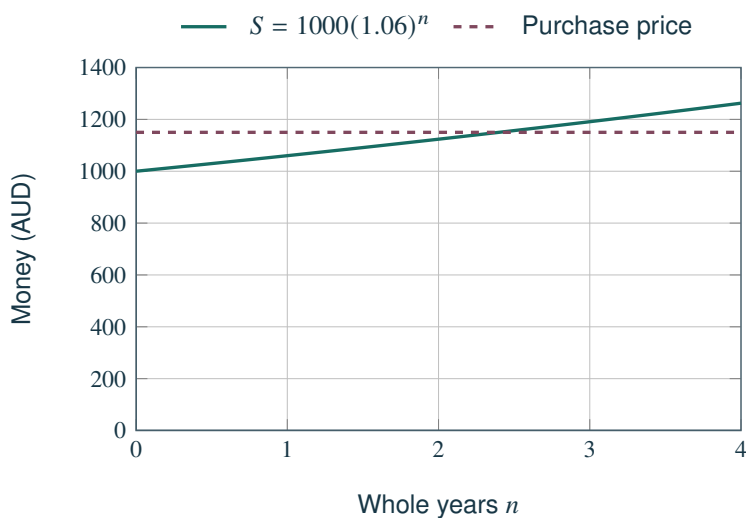
Exponential growth and decay

Name: _____ Class: _____ Date: _____

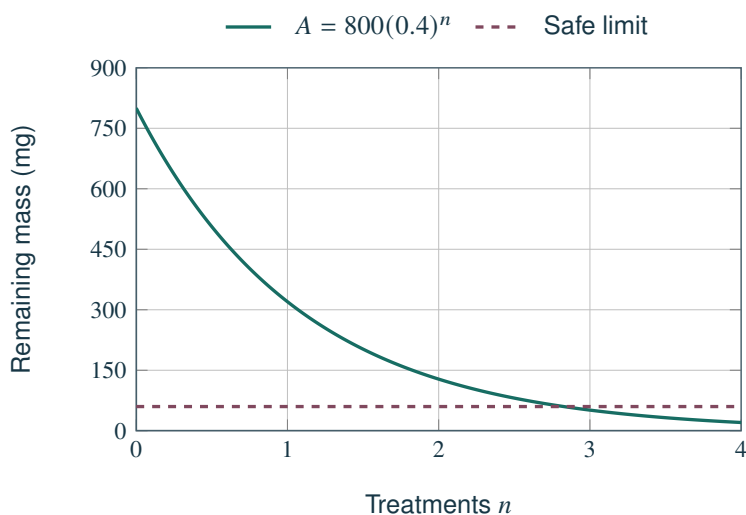
Use your classroom lesson if needed. Show your working; label all graphs.

1. Write the multipliers for a 6% increase and a 22% decrease.

2. A \$1,600 printer loses 10% of its remaining value yearly. Write its model and value after three years.



Savings begin at \$1,000; the purchase price is \$1,150. Use this graph for Questions 3 and 4.



Contaminant amount and the 60 mg limit are supplied. Read whole treatments only.

3. The graph shows $S = 1000(1.06)^n$. Find the savings after two years, rounded to cents.

4. Using the same graph, find the first whole year when savings cover \$1,150. Give the balance in that year.

5. The sample follows $A = 800(0.4)^n$ mg. How much remains after two treatments? Is it at or below the 60 mg limit?

6. Using the same graph, find the first safe whole treatment cycle. State the amount at that cycle and one cycle earlier.

6N / Classroom answers

Teacher copy. Accept equivalent exact forms and valid alternative reasoning.

1. Percentage multipliers

1.09 and 0.86.

- $1 + 0.09 = 1.09$; $1 - 0.14 = 0.86$.
-

2. Write and use a model

$V = 1200(0.85)^n$; \$867.00.

- The multiplier is 0.85.
 - $V(2) = 1200(0.85)^2 = 867$.
-

3. Calculate compound growth

\$1,007.77.

- $S(3) = 800(1.08)^3 = 1007.7696$.
 - Round the final balance to cents.
-

4. Earliest affordable year

Year 4; \$1,088.39.

- Year 3 has $1007.7696 < 1050$.
 - Year 4 has $800(1.08)^4 = 1088.391168$, so it is the first affordable whole year.
-

5. Calculate compound decay

112.5 mg; not yet safe.

- $A(3) = 900(0.5)^3 = 112.5$.
 - $112.5 > 100$.
-

6. First safe whole cycle

Cycle 4: 56.25 mg; cycle 3: 112.5 mg.

- $A(4) = 900(0.5)^4 = 56.25 \leq 100$.
 - The previous cycle has $112.5 > 100$, so cycle 4 is first.
-

6N / Homework answers

Teacher copy. Accept equivalent exact forms and valid alternative reasoning.

1. Percentage multipliers

1.06 and 0.78.

- $1 + 0.06 = 1.06$; $1 - 0.22 = 0.78$.
-

2. Write and use a model

$V = 1600(0.9)^n$; \$1,166.40.

- The multiplier is 0.9.
 - $V(3) = 1600(0.9)^3 = 1166.4$.
-

3. Calculate compound growth

\$1,123.60.

- $S(2) = 1000(1.06)^2 = 1123.6$.
-

4. Earliest affordable year

Year 3; \$1,191.02.

- Year 2 has $1123.6 < 1150$.
 - Year 3 has $1000(1.06)^3 = 1191.016$, so it is the first affordable whole year.
-

5. Calculate compound decay

128 mg; not yet safe.

- $A(2) = 800(0.4)^2 = 128$.
 - $128 > 60$.
-

6. First safe whole cycle

Cycle 3: 51.2 mg; cycle 2: 128 mg.

- $A(3) = 800(0.4)^3 = 51.2 \leq 60$.
 - The previous cycle has $128 > 60$, so cycle 3 is first.
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