

CHAPTER 6 / 61

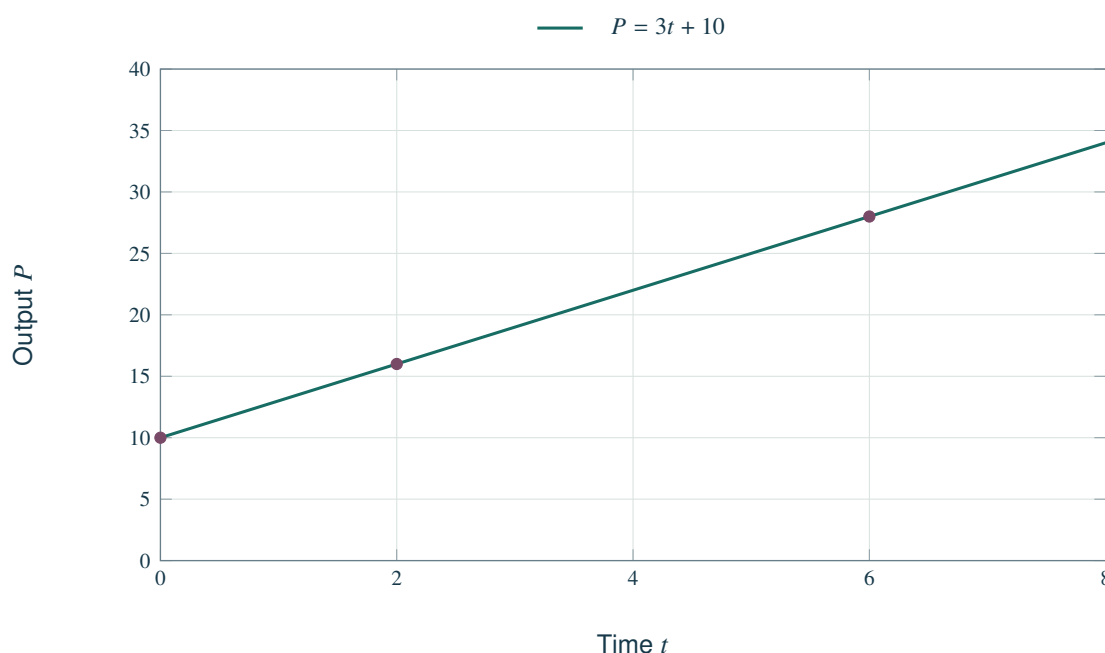
Linear modelling

Name: _____ Class: _____ Date: _____

Build a model from measured data, interpret its parameters and use it to make a practical decision. Check units and decide where a linear model remains sensible.

What you will learn

- Find a linear rule from two readings or a practical description.
- Interpret gradient and intercept using units and context.
- Use and compare models, including working backwards from a target.

See the mathematics

The starting output and rate have different roles in a linear model.

Key ideas

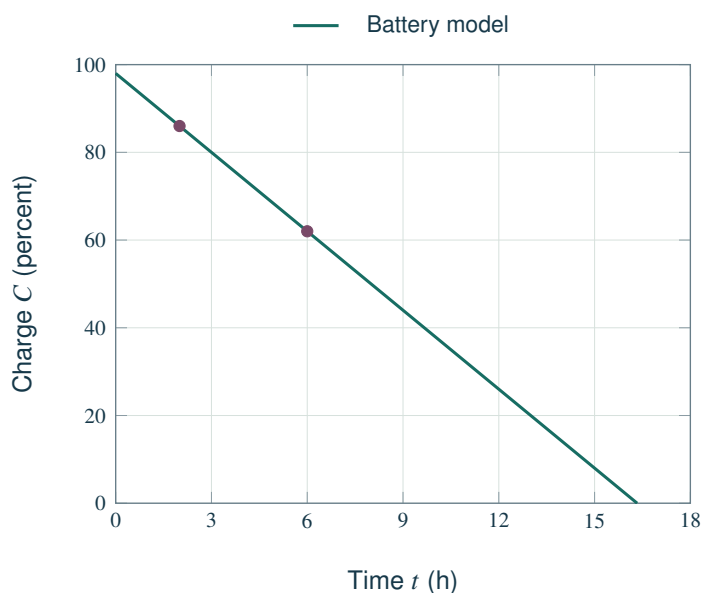
- For a non-vertical line, $m = \frac{y_2 - y_1}{x_2 - x_1}$. Keep the coordinate order consistent.
- Substitute a known point into $y = mx + c$ to find c . Check the second point.
- In context, the gradient is the change in output per unit of input; the intercept is the output when input is zero.
- A model is an assumption. Restrict its domain when negative time, volume or item counts would be impossible.

Pages 1–8: student resource. Pages 9–10: worked answers.

Worked examples / connect the calculation to the graph

1. Recover a battery model

A battery reports 86% charge after 2 hours and 62% after 6 hours. Model its percentage charge C after t hours.



$$1. m = \frac{62-86}{6-2} = -6 \text{ percentage points per hour.}$$

$$2. 86 = -6(2) + c, \text{ so } c = 98.$$

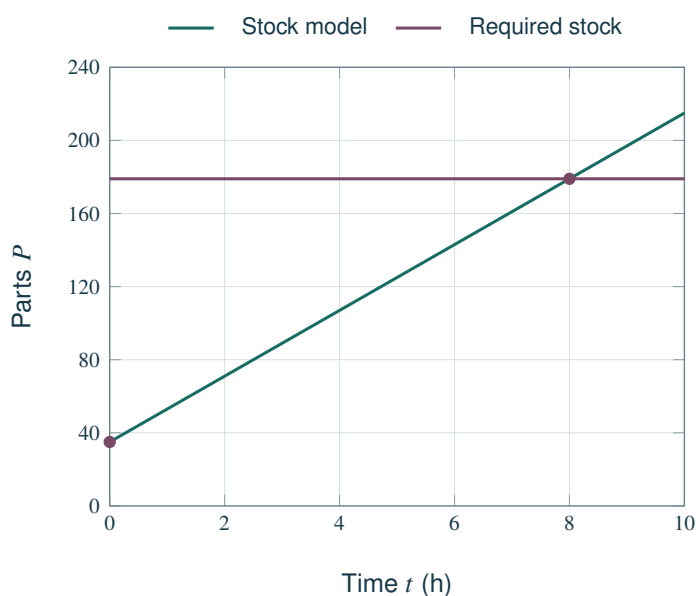
Result

$C = -6t + 98$, until the battery reaches 0%.

The complete worked battery model includes both observed readings and its physically meaningful endpoint.

2. Use a model backwards

A workshop already has 35 completed parts and produces 18 parts per hour. When will the stock reach 179 parts?



$$1. \text{ The stock model is } P = 18t + 35.$$

$$2. 179 = 18t + 35 \text{ gives } 144 = 18t.$$

Result

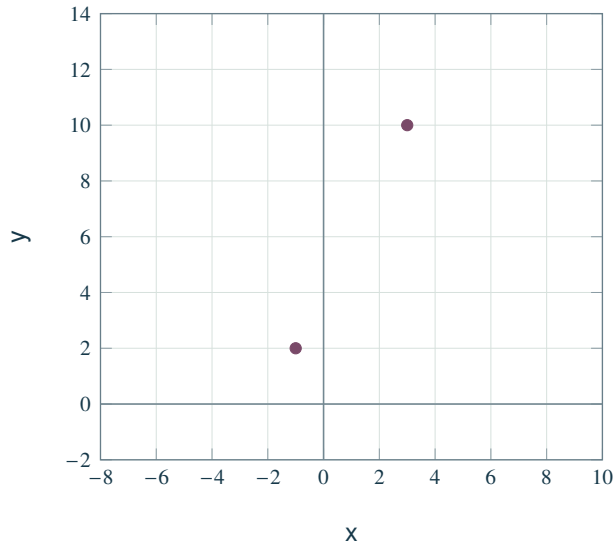
$t = 8$ hours.

The worked example uses the model backwards from the required stock level.

1. Recover the model

Use the graph to record your thinking. Show calculations and include units where needed.

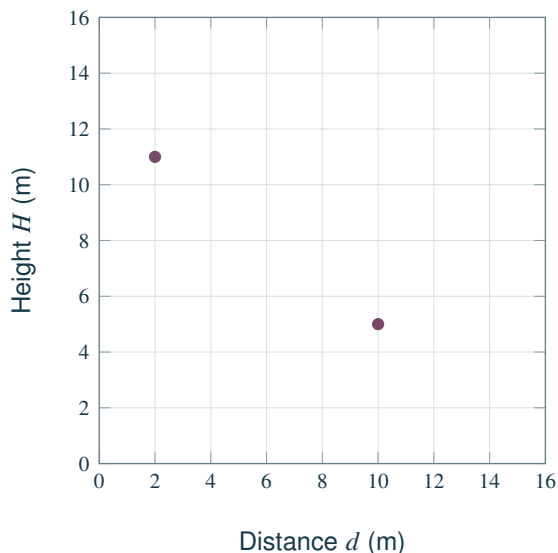
1. A straight route passes through $(-1, 2)$ and $(3, 10)$. Find its equation and check both points. Draw the route on the diagram.



WORKING / DECISION

Only the two supplied route points are shown. Construct the recovered line.

2. A cable height is 11 m at horizontal distance 2 m and 5 m at distance 10 m. Find the height model $H = md + c$. Plot the model through the two recorded points.



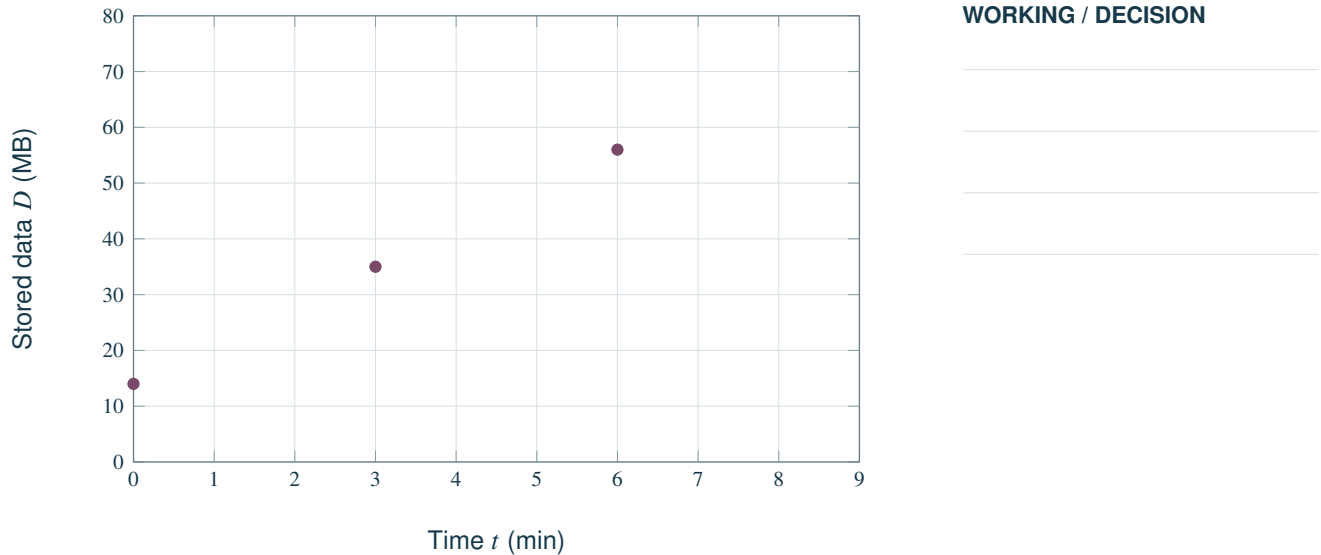
WORKING / DECISION

Measured cable heights. Add the model and label its starting-height prediction.

1. Recover the model

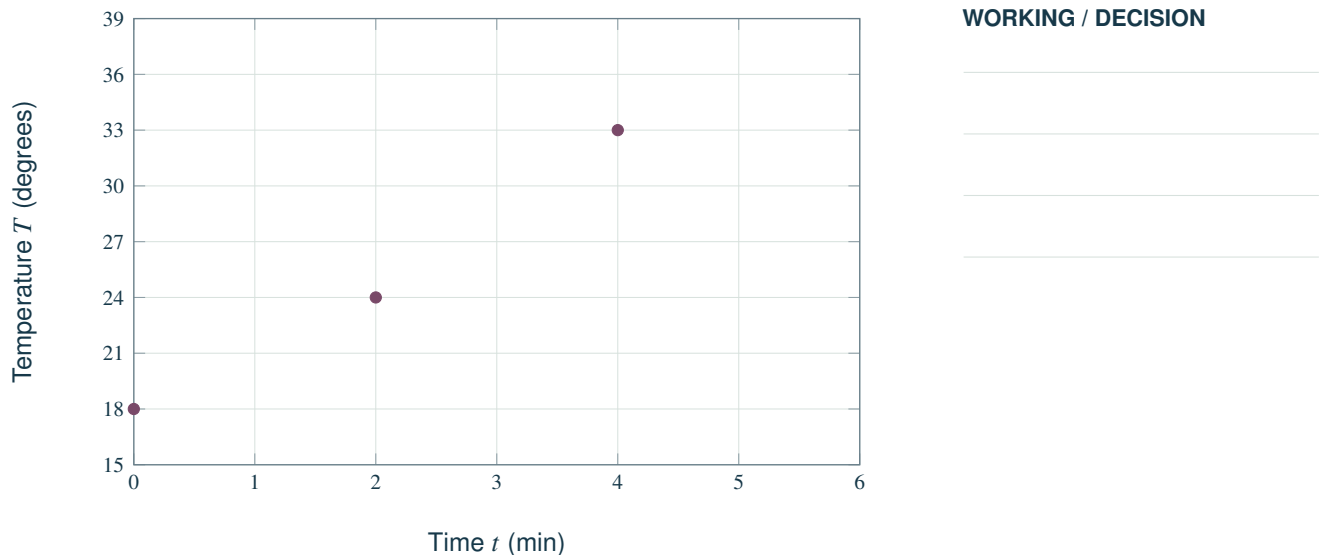
Use the graph to record your thinking. Show calculations and include units where needed.

3. A scanner gives these readings: time $t = 0, 3, 6$ minutes; stored data $D = 14, 35, 56$ MB. Decide whether a linear model fits and write it. Draw your proposed model through the graph of the readings.



All three supplied scanner readings are shown; use them to assess a single linear model.

4. A sensor gives (t, T) readings $(0, 18), (2, 24), (4, 33)$. Can one straight-line model fit all three exactly? Show the two interval rates and compare them with the plotted points.

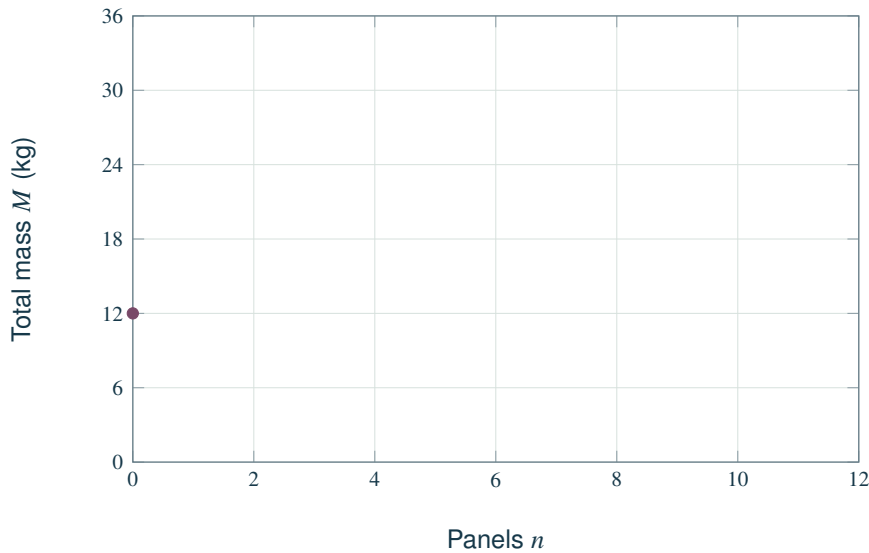


The graph contains only the recorded temperatures, not a fitted line.

2. Use the parameters

Use the graph to record your thinking. Show calculations and include units where needed.

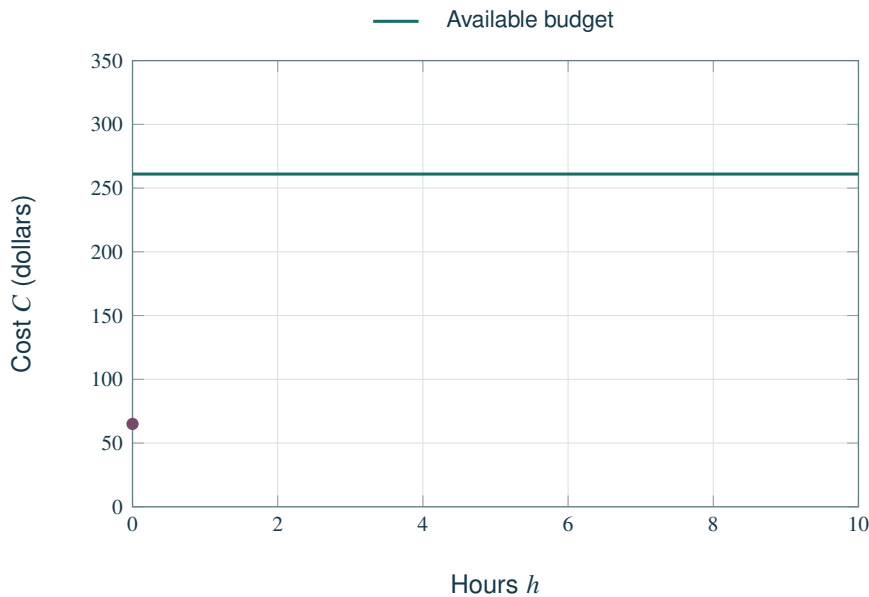
5. A display frame weighs 12 kg and each panel adds 1.75 kg. Write total mass M for n panels. Interpret both parameters and find the mass with 8 panels. Graph the model and mark the eight-panel configuration.



WORKING / DECISION

The empty frame is shown. Add the mass model and the requested configuration.

6. A maker charges a \$65 setup fee and \$28 per hour. Write C in terms of h . A customer has \$261. How many hours can they buy? Construct the cost model and show the purchase on the graph.



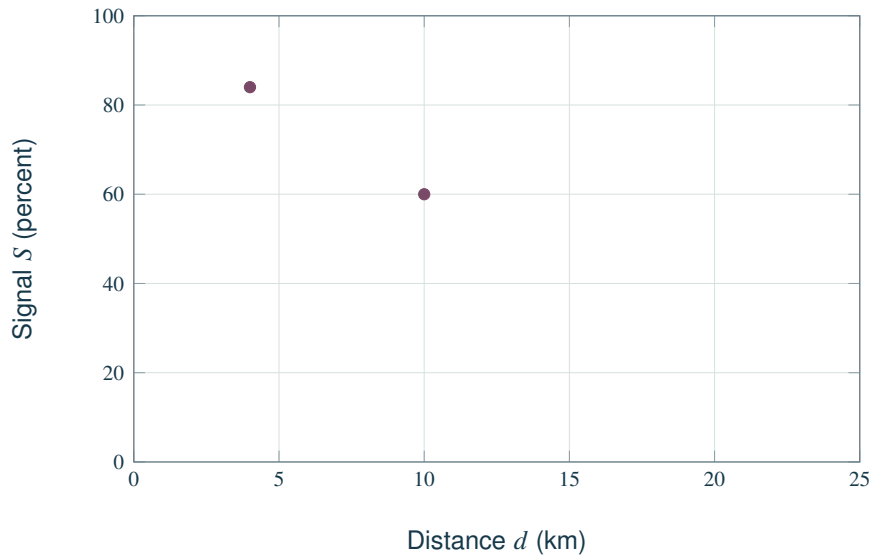
WORKING / DECISION

The setup fee and available budget are supplied. Add the maker cost model.

2. Use the parameters

Use the graph to record your thinking. Show calculations and include units where needed.

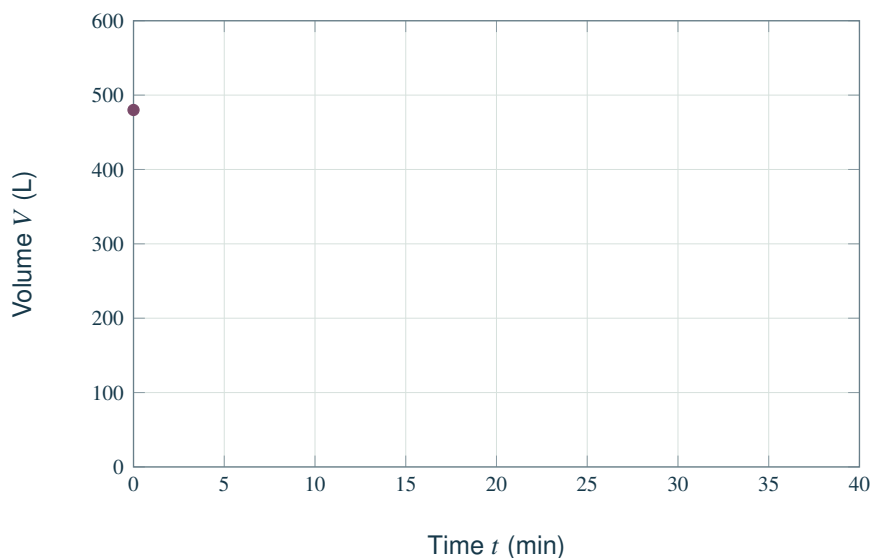
7. A rover's signal is 84% at 4 km and 60% at 10 km. Find $S = md + c$ using percentage units. Predict the percentage signal at 15 km and add that prediction to the graph of the supplied readings.



WORKING / DECISION

Two measured signal strengths are given. Construct the percentage model and the requested forecast.

8. Water volume follows $V = 480 - 16t$, where V is litres and t is minutes after a valve opens. Interpret both numbers, find when the tank empties and state a sensible domain. Draw the physically meaningful part of the graph.



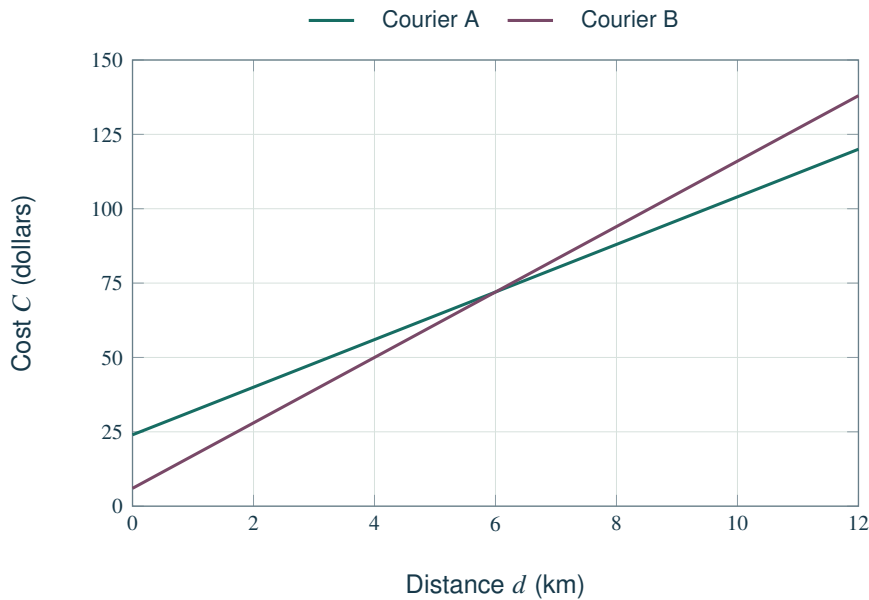
WORKING / DECISION

The initial tank volume is supplied. Draw the model only while its physical interpretation is valid.

3. Make the decision

Use the graph to record your thinking. Show calculations and include units where needed.

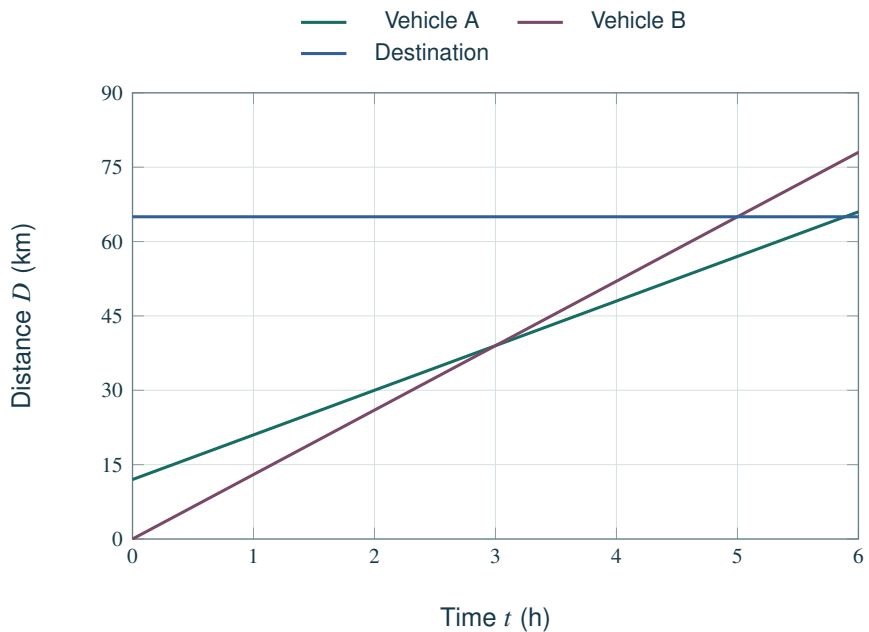
9. Courier A charges $C = 8d + 24$ and Courier B charges $C = 11d + 6$, in dollars for d kilometres. Find the distance at which the costs match, and choose the cheaper courier for 10 km. Mark the comparison on the graph.



WORKING / DECISION

The two supplied price rules are plotted for comparison. Calculate and mark the matching point.

10. Rescue vehicles travel on the same road. A is at $D = 9t + 12$ km and B at $D = 13t$ km after t hours. Find when and where B catches A. Which reaches the 65 km point first? Use the common graph to support your decision.



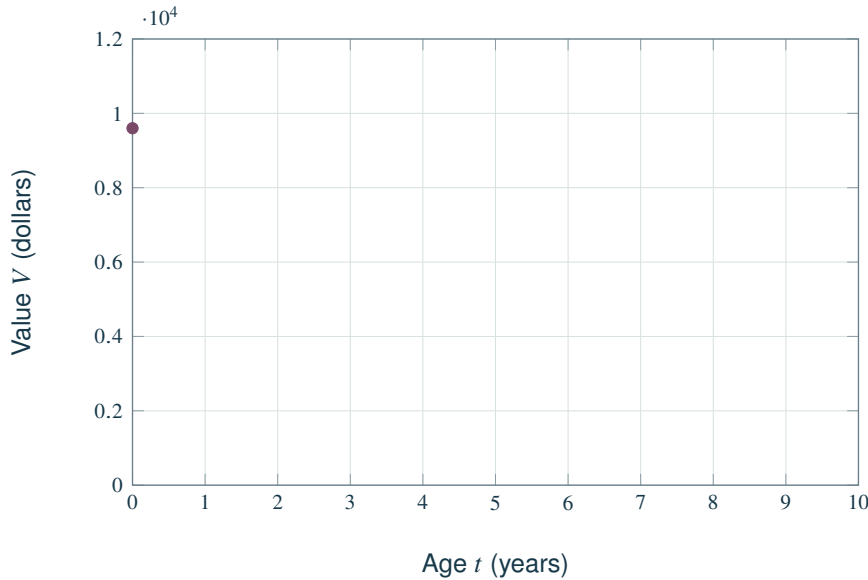
WORKING / DECISION

The supplied distance models and common destination are drawn on one set of axes.

3. Make the decision

Use the graph to record your thinking. Show calculations and include units where needed.

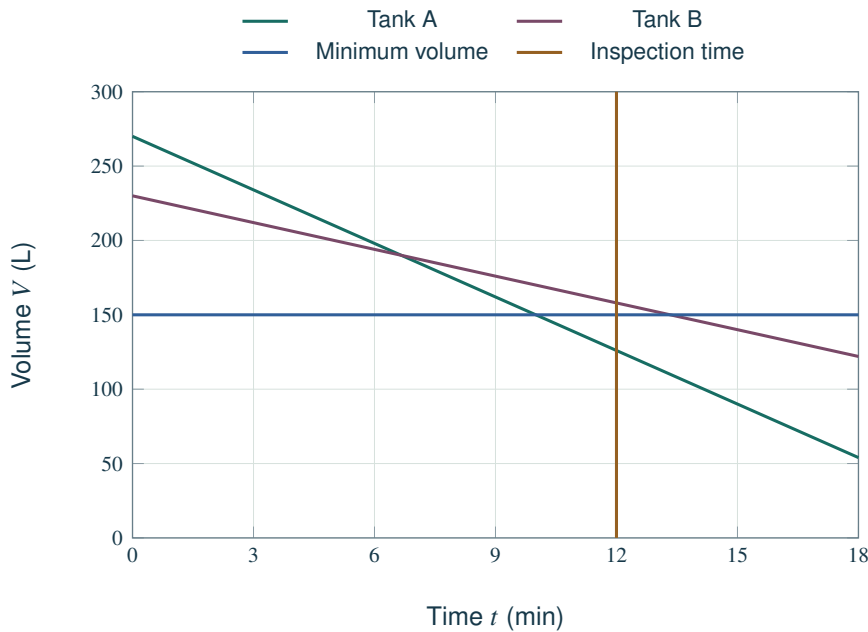
11. A machine costs \$9,600 and loses \$1,200 of value each year under a straight-line model. Write $V(t)$, find its value after 5 years and explain why the model should not continue unchanged past 8 years. Show the valid model on the graph.



WORKING / DECISION

Only the original machine value is given. Construct the depreciation model over a sensible domain.

12. A water station must hold at least 150 L after 12 minutes. Tank A follows $V = 270 - 12t$ and Tank B follows $V = 230 - 6t$. Choose a suitable tank, state its remaining volume and find how many minutes it can operate before reaching 150 L. Mark your decision on the comparison graph.



WORKING / DECISION

The two given tank models, inspection time and minimum operating volume are shown.

6I / Worked answers

Teacher appendix. Equivalent exact forms are accepted. Use the reasoning to check student work.

1. $y = 2x + 4$.

- $m = \frac{10-2}{3-(-1)} = \frac{8}{4} = 2$.
 - $2 = 2(-1) + c$ gives $c = 4$. Checks: 2 and 10.
-

2. $H = -\frac{3}{4}d + \frac{25}{2}$.

- $m = \frac{5-11}{10-2} = -\frac{6}{8} = -\frac{3}{4}$.
 - $11 = -\frac{3}{4}(2) + c$, so $c = 12.5$.
-

3. Yes; $D = 7t + 14$.

- The changes are 21 MB in each 3-minute interval, a constant 7 MB/min.
 - At $t = 0$ the stored data is 14 MB.
-

4. No; the rates are 3 and 4.5 degrees per minute.

- First rate $= \frac{24-18}{2} = 3$.
 - Second rate $= \frac{33-24}{2} = 4.5$. Unequal rates rule out one exact line.
-

5. $M = 1.75n + 12$; 26 kg for 8 panels.

- 1.75 kg/panel is the added mass; 12 kg is the empty frame.
 - $M = 1.75(8) + 12 = 26$.
-

6. $C = 28h + 65$; 7 hours.

- Set $261 = 28h + 65$.
 - $196 = 28h$ gives $h = 7$.
-

6I / Worked answers

Teacher appendix. Equivalent exact forms are accepted. Use the reasoning to check student work.

7. $S = -4d + 100$; 40% at 15 km.

- $m = \frac{60-84}{10-4} = -4$ percentage points/km; $c = 100$.
- $S(15) = -60 + 100 = 40$. A proportion model would instead be $S = -0.04d + 1$.

8. Starts with 480 L; loses 16 L/min; empty at 30 min; $0 \leq t \leq 30$.

- Set $V = 0$: $16t = 480$, so $t = 30$.
- Beyond 30 minutes the line predicts negative volume, which is not physical.

9. Match at 6 km (\$72); Courier A is cheaper at 10 km (\$104 versus \$116).

- $8d + 24 = 11d + 6$ gives $18 = 3d$.
- Evaluate both at $d = 10$.

10. Meet at 3 h, 39 km. B reaches 65 km first, at 5 h.

- $9t + 12 = 13t$ gives $t = 3$ and $D = 39$.
- A needs $\frac{65-12}{9} = \frac{53}{9}$ h; B needs $\frac{65}{13} = 5$ h.

11. $V = 9600 - 1200t$; \$3,600 after 5 years; value reaches zero at 8 years.

- $V(5) = 9600 - 6000 = 3600$.
- At $t > 8$ the rule gives negative value, so the physical interpretation fails.

12. Tank B; 158 L after 12 minutes; reaches 150 L at $\frac{40}{3}$ minutes.

- $A(12) = 126 < 150$, while $B(12) = 158 \geq 150$.
- $150 = 230 - 6t$ gives $t = \frac{80}{6} = \frac{40}{3}$ minutes (13 min 20 s).