

CHAPTER 6 / 6H

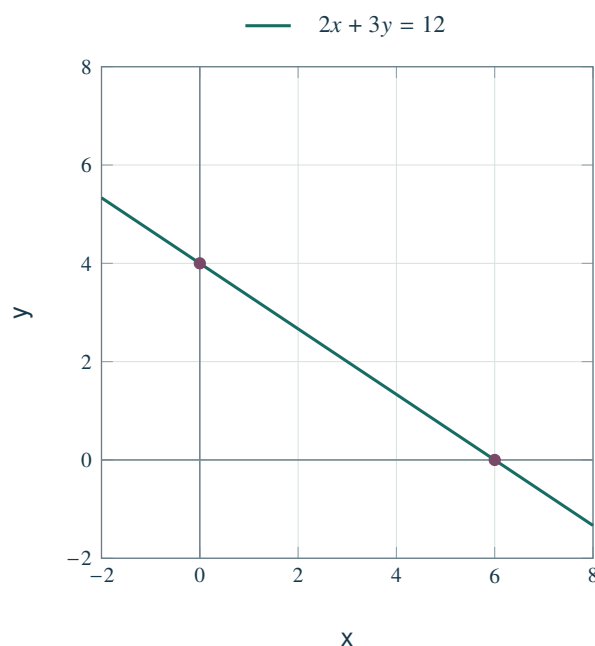
Graphing straight lines using intercepts

Name: _____ Class: _____ Date: _____

Find where a line crosses each axis and use those points to construct the graph. Recognise when the intercept method needs another point.

What you will learn

- Find x - and y -intercepts by setting the other variable to zero.
- Sketch lines from intercepts, including fractional and negative values.
- Interpret intercepts in a context and handle special cases.

See the mathematics

The line is fixed by two different axis intercepts. Each intercept has one zero coordinate.

Key ideas

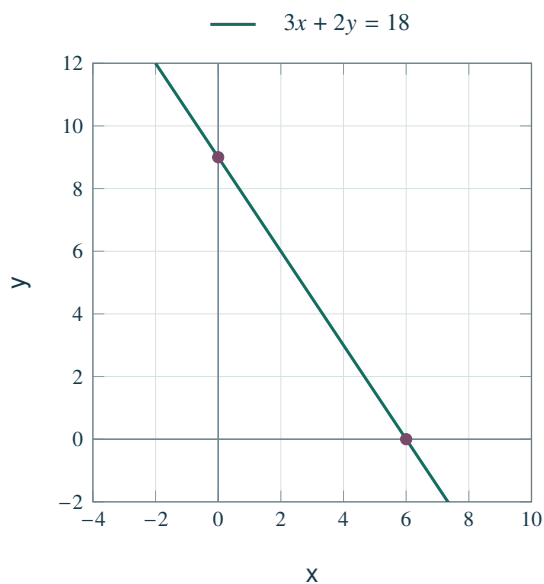
- At an x -intercept $y = 0$; at a y -intercept $x = 0$.
- For $ax + by = d$ with $a \neq 0$ and $b \neq 0$, the intercepts are $\left(\frac{d}{a}, 0\right)$ and $\left(0, \frac{d}{b}\right)$.
- Two different points determine a straight line. If both intercepts are $(0, 0)$, find another point.
- A horizontal or vertical line may miss an axis. Only the non-negative part of a graph may make sense in a physical context.

Pages 1–8: student resource. Pages 9–10: worked answers.

Worked examples / connect the calculation to the graph

1. Construct from two intercepts

Find the intercepts of $3x + 2y = 18$.



1. Set $y = 0$: $3x = 18$, giving $x = 6$.

2. Set $x = 0$: $2y = 18$, giving $y = 9$.

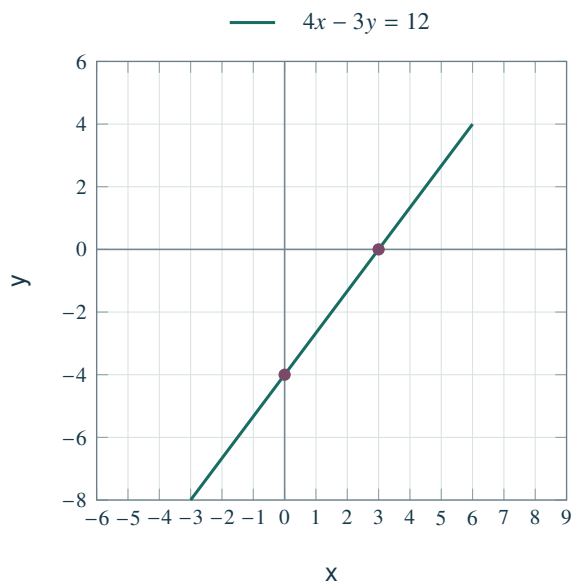
Result

Plot $(6, 0)$ and $(0, 9)$, then draw the straight line through them.

The worked example marks both calculated intercepts and the resulting straight line.

2. Work with negative values

Find the intercepts of $4x - 3y = 12$.



1. With $y = 0$, $4x = 12$, so $x = 3$.

2. With $x = 0$, $-3y = 12$, so $y = -4$.

Result

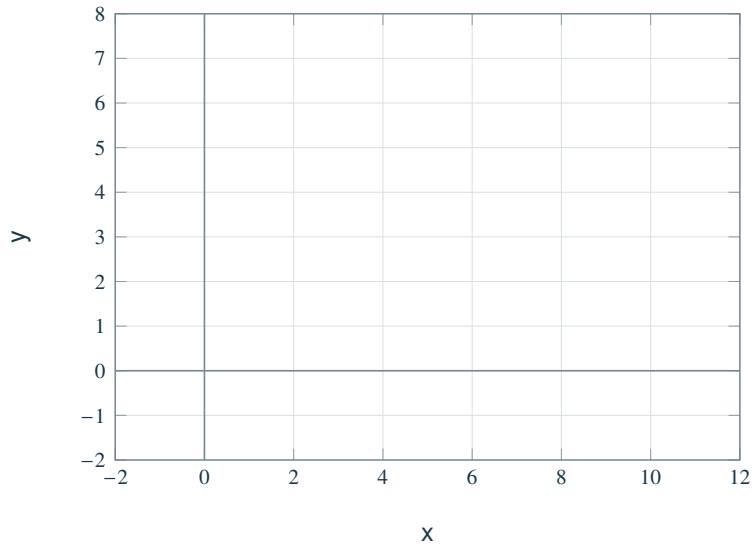
The intercepts are $(3, 0)$ and $(0, -4)$.

One intercept lies on the negative half of its axis in this worked example.

1. Find the crossing points

Use the graph to record your thinking. Show calculations and include units where needed.

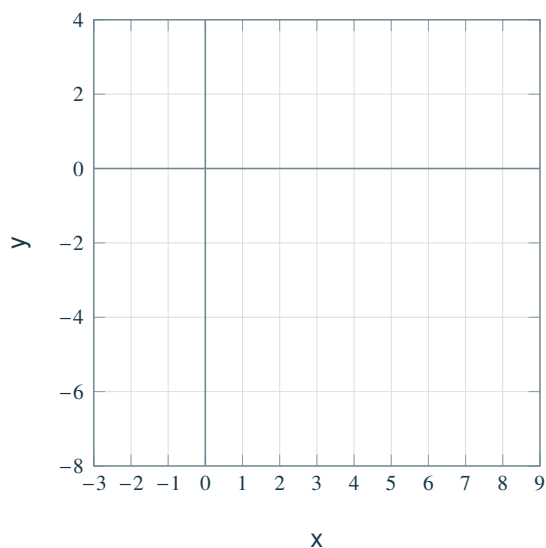
1. Find both intercepts of $2x + 5y = 20$. Explain why the y -coordinate of the x -intercept must be zero. Plot your intercepts and draw the line.



WORKING / DECISION

Construction grid: calculate and plot the two intercepts before joining them.

2. Find both intercepts of $3x - 2y = 12$. In which half of the y -axis does the line cross? Draw the line through your calculated intercepts.



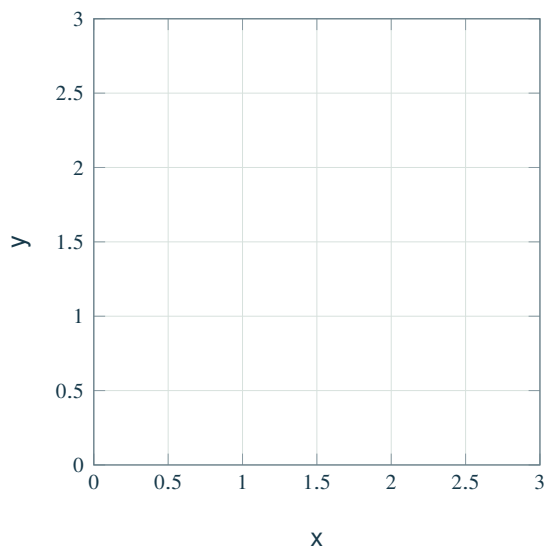
WORKING / DECISION

Construction grid with positive and negative axis values.

1. Find the crossing points

Use the graph to record your thinking. Show calculations and include units where needed.

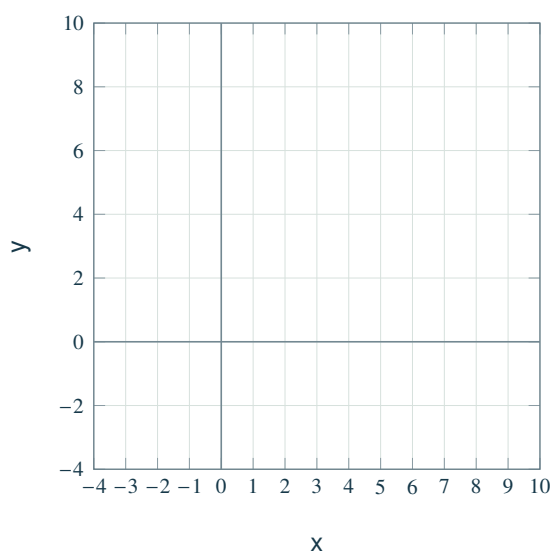
3. Find both intercepts of $4x + 6y = 9$. Leave answers as exact fractions. Mark the intercepts accurately on the half-unit grid and draw the line.



WORKING / DECISION

Construction grid: some required coordinates lie between marked half-unit ticks.

4. Find both intercepts of $y = -\frac{3}{2}x + 6$. Check the x -intercept in the original equation, then draw the line.



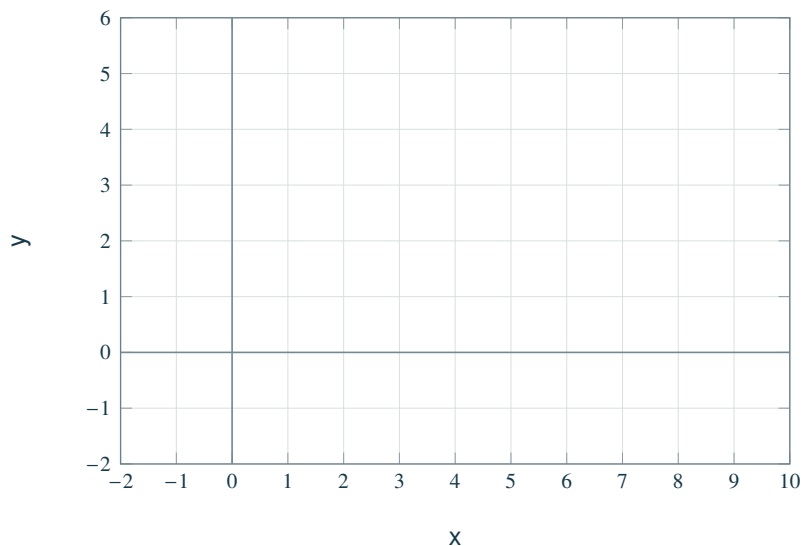
WORKING / DECISION

Construction grid for intercepts obtained from gradient-intercept form.

2. Plot and handle exceptions

Use the graph to record your thinking. Show calculations and include units where needed.

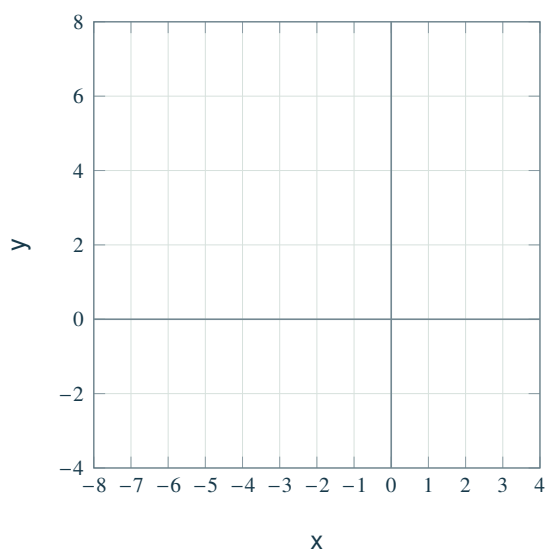
5. Use intercepts to sketch $x + 2y = 8$. State where the line crosses each axis and label the line.



WORKING / DECISION

Construction grid: mark and label both intercepts.

6. Use intercepts to sketch $2x - y = -6$. Give a third point to check the graph.



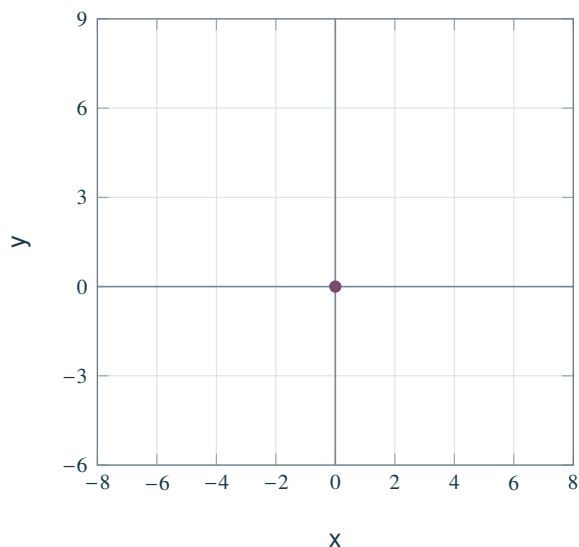
WORKING / DECISION

Construction grid for both intercepts and an independently checked third point.

2. Plot and handle exceptions

Use the graph to record your thinking. Show calculations and include units where needed.

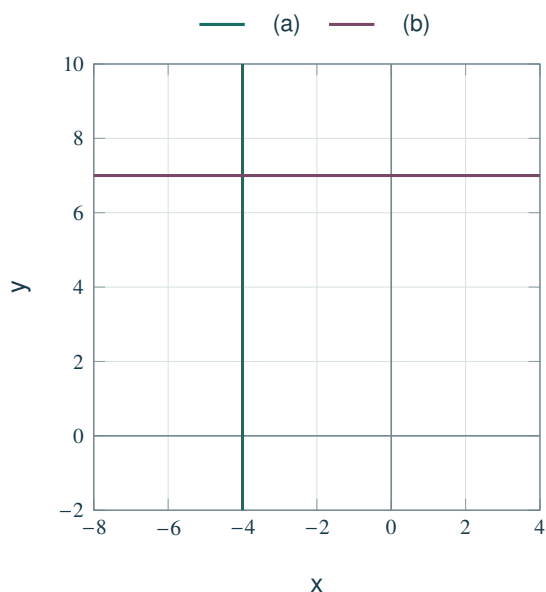
7. A student finds both intercepts of $y = 3x$ to be $(0, 0)$ and says a unique line can now be drawn. Explain what is missing and give a useful second point. Add your second point and the line to the diagram.



WORKING / DECISION

Only the shared intercept is provided. Add enough information to determine the line.

8. State the intercepts, if any, of (a) $x = -4$ and (b) $y = 7$. Explain why each line misses one axis. Mark the existing intercepts on the reference graph.



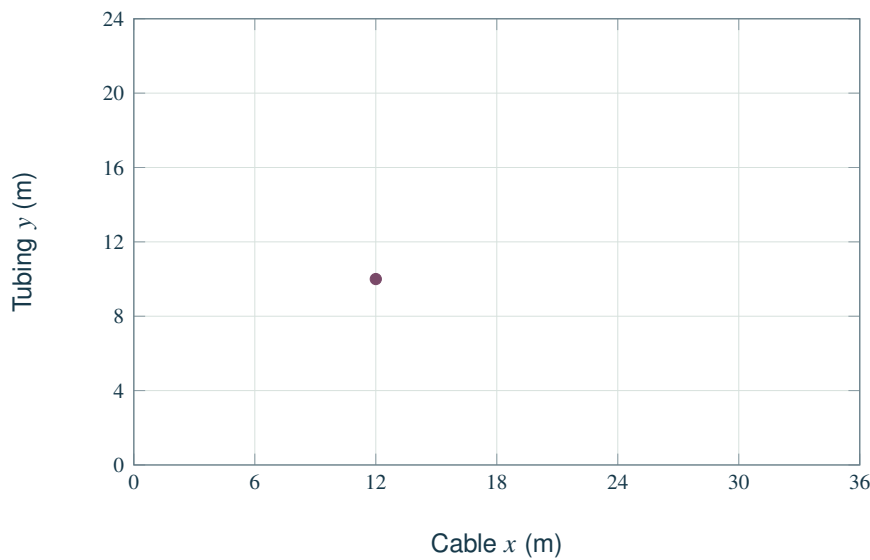
WORKING / DECISION

Reference graph of the two supplied special lines.

3. Use the intercepts

Use the graph to record your thinking. Show calculations and include units where needed.

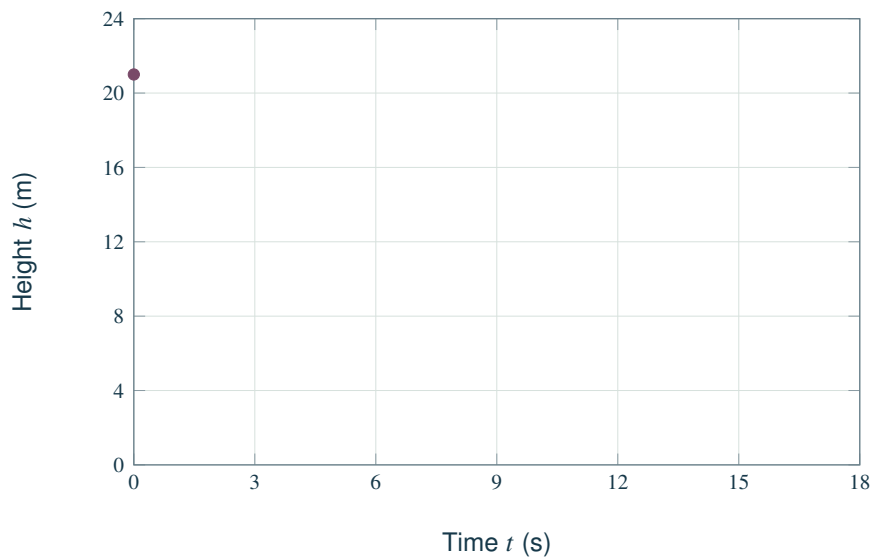
9. A workshop has \$180 for cable at \$6 per metre and tubing at \$9 per metre. Its full-budget boundary is $6x + 9y = 180$. Find and interpret both intercepts. Is (12, 10) affordable? Draw the boundary and show the proposed purchase.



WORKING / DECISION

The proposed purchase is marked. Construct the full-budget boundary from your intercepts.

10. A lift's height is $h = 21 - 1.5t$, with h in metres and t in seconds. Find the two intercepts and explain each one. Give the time interval before the lift reaches ground level and draw that part of the journey.



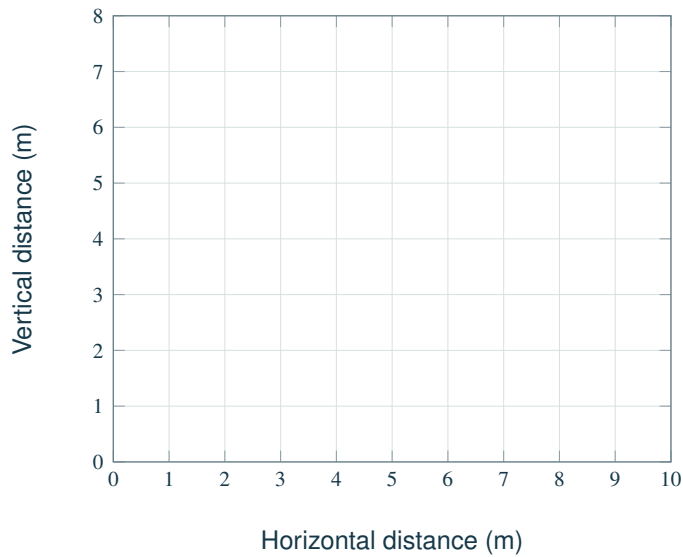
WORKING / DECISION

The starting height is given. Construct the journey only over its physical time interval.

3. Use the intercepts

Use the graph to record your thinking. Show calculations and include units where needed.

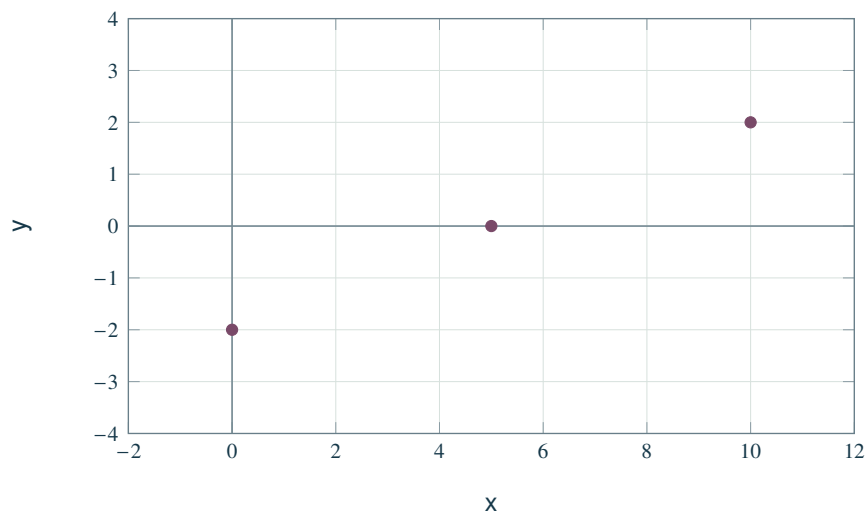
11. A triangular work zone has boundaries $x = 0$, $y = 0$ and $3x + 4y = 24$, with distances in metres. Find the two non-origin vertices and the area. Draw and shade the work zone on the grid.



WORKING / DECISION

Construction grid: use the axes and your calculated boundary to form the work zone.

12. A navigation boundary must pass through $(5, 0)$ and $(0, -2)$. Write its equation in the form $ax + by = d$ with integer coefficients, then check whether $(10, 2)$ lies on it. Draw the boundary through the supplied intercepts.



WORKING / DECISION

The required intercepts and proposed check point are supplied. Add the boundary.

6H / Worked answers

Teacher appendix. Equivalent exact forms are accepted. Use the reasoning to check student work.

1. x -intercept $(10, 0)$; y -intercept $(0, 4)$.

- $y = 0$ gives $2x = 20$. $x = 0$ gives $5y = 20$.
 - Every point on the x -axis has y -coordinate 0.
-

2. $(4, 0)$ and $(0, -6)$; negative half of the y -axis.

- Set $y = 0$: $x = \frac{12}{3} = 4$.
 - Set $x = 0$: $y = \frac{12}{-2} = -6$.
-

3. $(\frac{9}{4}, 0)$ and $(0, \frac{3}{2})$.

- At $y = 0$, $x = \frac{9}{4}$.
 - At $x = 0$, $y = \frac{9}{6} = \frac{3}{2}$.
-

4. $(4, 0)$ and $(0, 6)$.

- $0 = -\frac{3}{2}x + 6$ gives $x = 4$.
 - Substituting gives $-\frac{3}{2}(4) + 6 = 0$.
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5. Straight line through $(8, 0)$ and $(0, 4)$.

- $y = 0$ gives $x = 8$; $x = 0$ gives $y = 4$.
 - Join and extend through both points.
-

6. Line through $(-3, 0)$, $(0, 6)$; another point is $(-2, 2)$.

- $y = 0$ gives $x = -3$; $x = 0$ gives $y = 6$.
 - At $x = -2$, $-4 - y = -6$ gives $y = 2$.
-

6H / Worked answers

Teacher appendix. Equivalent exact forms are accepted. Use the reasoning to check student work.

7. The two intercepts are the same point; use another point such as $(1, 3)$.

- A single point cannot determine the direction of a line.
 - Set $x = 1$ to obtain $y = 3$, then join $(0, 0)$ and $(1, 3)$.
-

8. (a) $(-4, 0)$, no y -intercept. (b) $(0, 7)$, no x -intercept.

- $x = -4$ can never have $x = 0$.
 - $y = 7$ can never have $y = 0$.
-

9. $(30, 0)$: all cable; $(0, 20)$: all tubing. $(12, 10)$ costs \$162, so is affordable.

- $\frac{180}{6} = 30$ and $\frac{180}{9} = 20$.
 - $6(12) + 9(10) = 72 + 90 = 162 \leq 180$.
-

10. $(0, 21)$: initial height; $(14, 0)$: reaches ground; $0 \leq t \leq 14$.

- At $t = 0$, $h = 21$.
 - Set $h = 0$: $1.5t = 21$, so $t = 14$.
-

11. Vertices $(8, 0)$, $(0, 6)$; area 24 m^2 .

- The intercept lengths are $\frac{24}{3} = 8$ and $\frac{24}{4} = 6$.
 - Area = $\frac{1}{2} \times 8 \times 6 = 24$.
-

12. $2x - 5y = 10$; $(10, 2)$ lies on it.

- Gradient = $\frac{0 - (-2)}{5 - 0} = \frac{2}{5}$, so $y = \frac{2}{5}x - 2$.
 - Multiply by 5 and rearrange. At $(10, 2)$, $20 - 10 = 10$.
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