

CHAPTER 6 / 6G

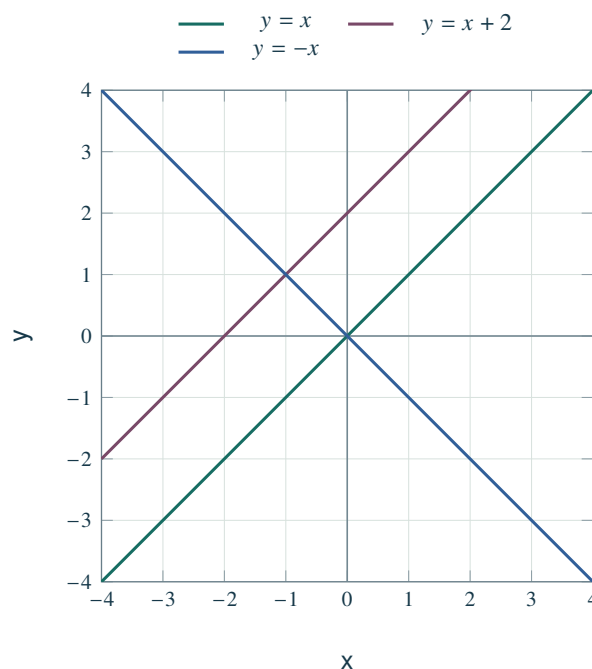
Parallel lines and perpendicular lines

Name: _____ Class: _____ Date: _____

Use gradients to plan parallel routes and right-angle crossings. Check the direction and the required point separately before approving a line.

What you will learn

- Find gradients from equations and pairs of points.
- Construct parallel and perpendicular lines through required points.
- Explain negative reciprocals and horizontal/vertical exceptions.

See the mathematics

Equal-scale axes preserve the visual right angle. The two rising lines are parallel.

Key ideas

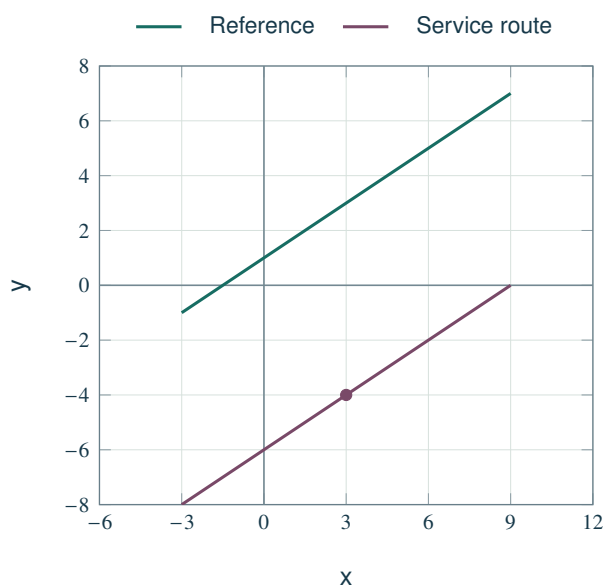
- Distinct parallel non-vertical lines have the same gradient and different intercepts.
- For two non-zero finite gradients, perpendicular lines satisfy $m_1 m_2 = -1$.
- The negative reciprocal of $\frac{p}{q}$ is $-\frac{q}{p}$; change both the sign and the fraction orientation.
- A horizontal line is perpendicular to a vertical line. Do not use division by zero in the reciprocal rule.

Pages 1–8: student resource. Pages 9–10: worked answers.

Worked examples / connect the calculation to the graph

1. A parallel service route

Find the line parallel to $y = \frac{2}{3}x + 1$ that passes through $(3, -4)$.



Reference road and the completed parallel service route.

1. Keep $m = \frac{2}{3}$ and use $y = \frac{2}{3}x + c$.

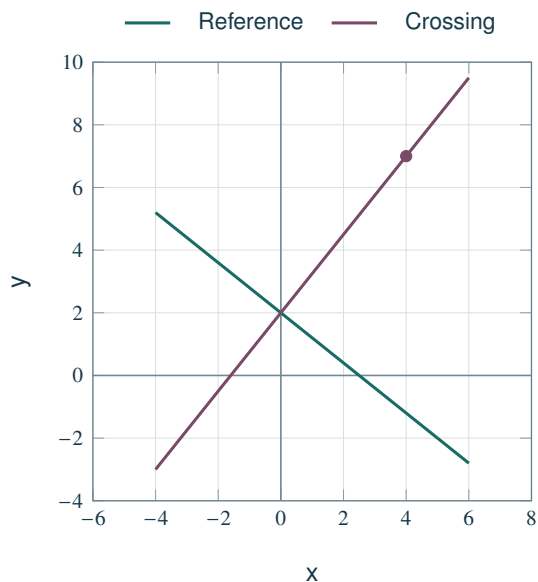
2. $-4 = 2 + c$, so $c = -6$.

Result

$$y = \frac{2}{3}x - 6.$$

2. A right-angle crossing

Find the line perpendicular to $y = -\frac{4}{5}x + 2$ through $(4, 7)$.



Completed crossing on equal-scale axes; both lines and the required point are shown.

1. The negative reciprocal of $-\frac{4}{5}$ is $\frac{5}{4}$.

2. $7 = \frac{5}{4}(4) + c = 5 + c$, so $c = 2$.

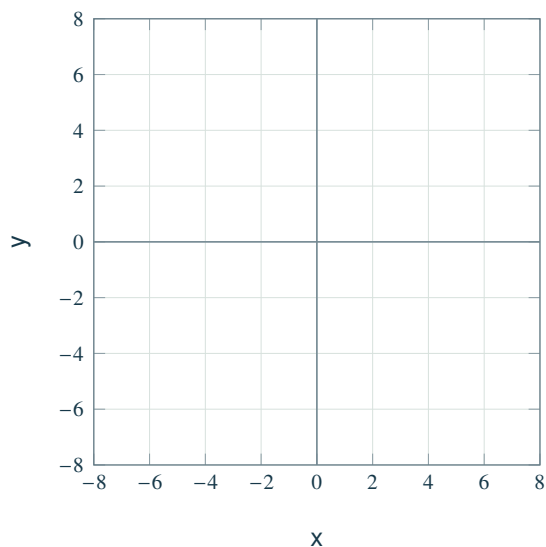
Result

$$y = \frac{5}{4}x + 2; \left(-\frac{4}{5}\right)\left(\frac{5}{4}\right) = -1.$$

1. Compare gradients

Use the graph to record your thinking. Show calculations and include units where needed.

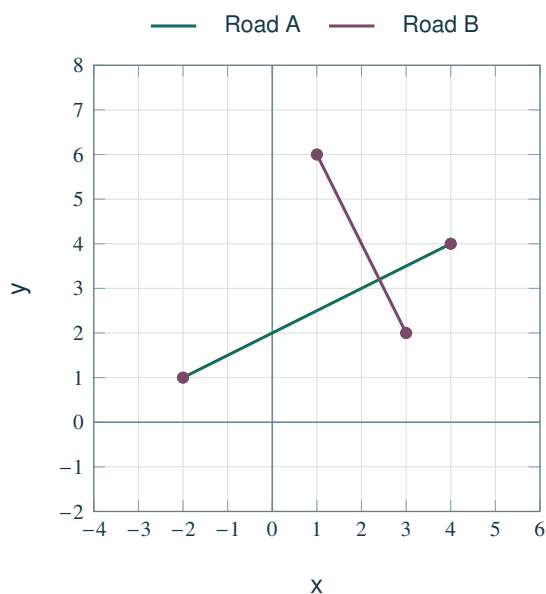
1. Classify each gradient pair as parallel, perpendicular or neither: (a) $\frac{3}{5}$ and $\frac{3}{5}$; (b) $-\frac{2}{7}$ and $\frac{7}{2}$; (c) 4 and -4 . Use the grid to sketch and label an example for each pair.



WORKING / DECISION

Construction grid for labelled examples of the three gradient pairs.

2. Road A passes through $(-2, 1)$ and $(4, 4)$. Road B passes through $(1, 6)$ and $(3, 2)$. Decide whether their directions are perpendicular. Use the equal-scale graph to check your calculation.



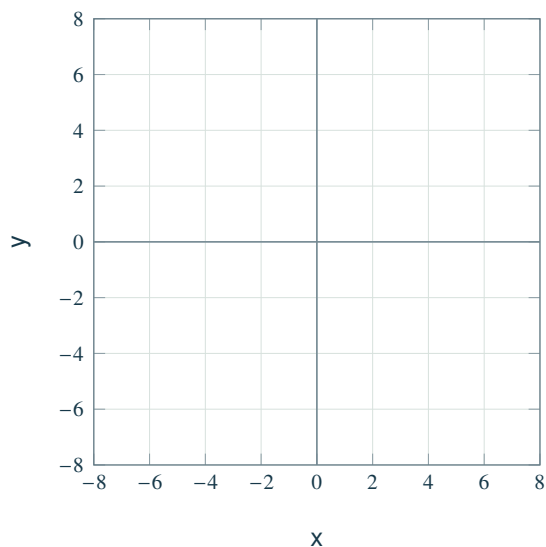
WORKING / DECISION

A joins its two supplied points; B joins its own supplied points. Both axes use the same unit scale.

1. Compare gradients

Use the graph to record your thinking. Show calculations and include units where needed.

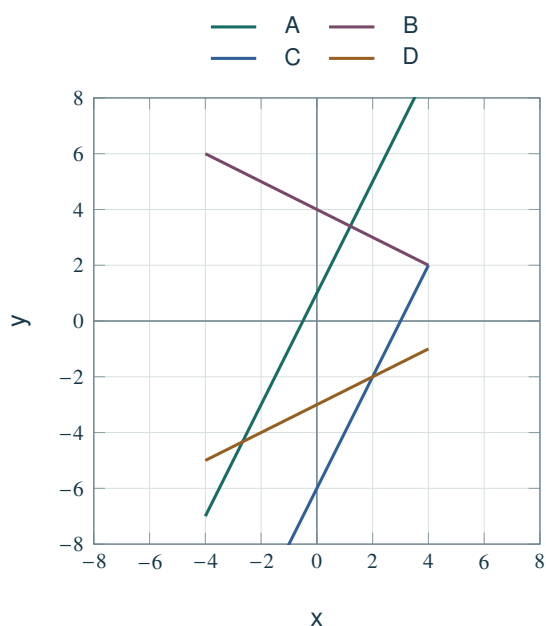
3. Write the perpendicular gradient for each: (a) $\frac{5}{8}$; (b) -3 ; (c) -0.25 . Check each answer by multiplication. On the grid, sketch each supplied direction and its proposed perpendicular through the origin.



WORKING / DECISION

Construction grid: label each direction pair a, b or c.

4. Lines A, B, C and D have equations $y = 2x + 1$, $y = -\frac{1}{2}x + 4$, $y = 2x - 6$ and $y = \frac{1}{2}x - 3$. List every parallel pair and every perpendicular pair. Match your decisions to the labelled graph.



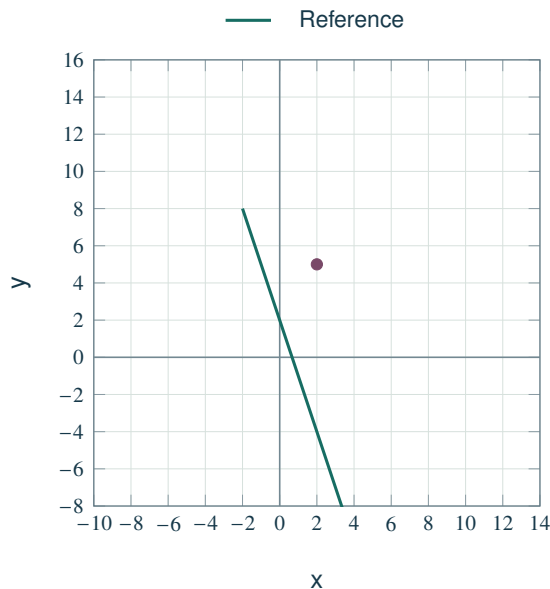
WORKING / DECISION

A, B, C and D match the equations in the stated order.

2. Build the routes

Use the graph to record your thinking. Show calculations and include units where needed.

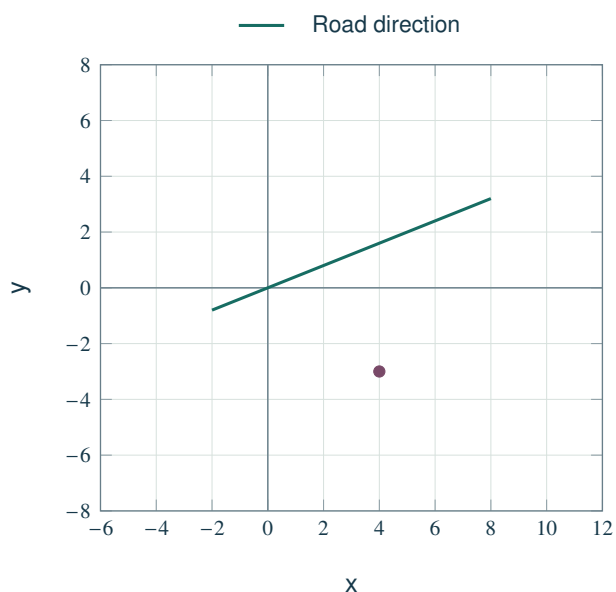
5. Find the equation of a line parallel to $y = -3x + 2$ through $(2, 5)$. Verify that your line passes through the point, then construct it on the diagram.



WORKING / DECISION

The reference line and required point are given. Add the new parallel line.

6. An emergency track must meet a road of gradient $\frac{2}{5}$ at a right angle and pass through $(4, -3)$. Find the emergency track equation and draw it on the diagram.



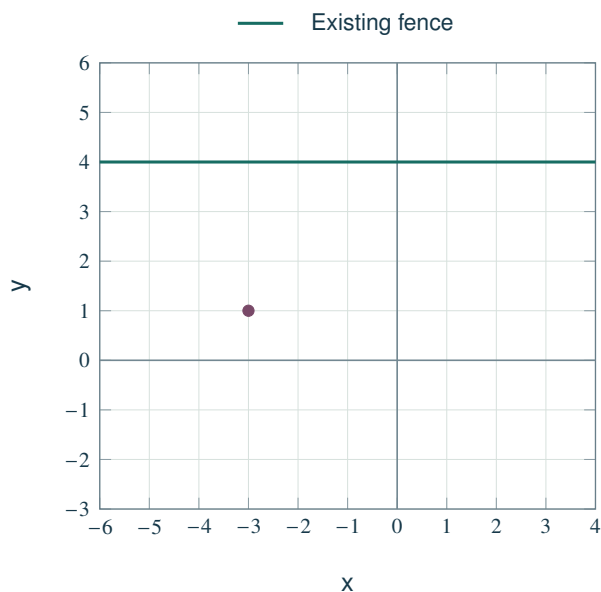
WORKING / DECISION

The supplied road direction and the required point. Construct the emergency track.

2. Build the routes

Use the graph to record your thinking. Show calculations and include units where needed.

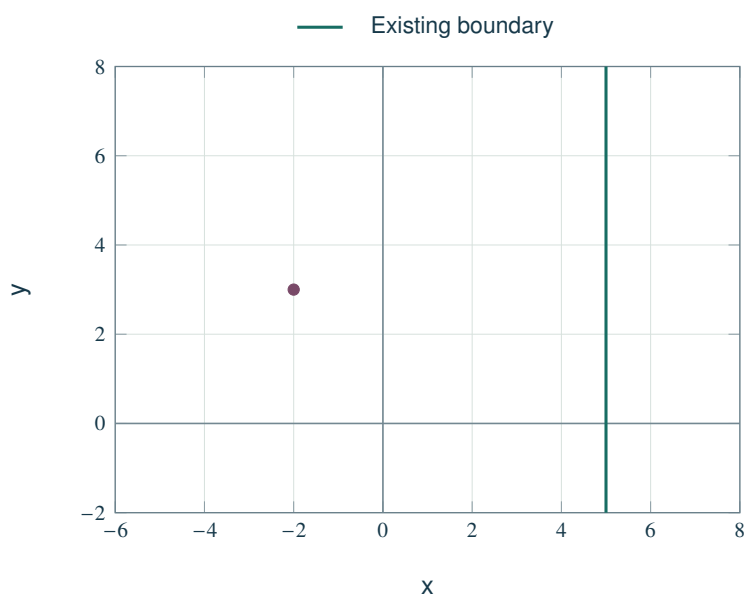
7. A fence follows $y = 4$. Draw the perpendicular fence through $(-3, 1)$. State its equation and explain why no finite m works.



WORKING / DECISION

The existing horizontal fence and required point; add the new fence.

8. A boundary follows $x = 5$. Give both a parallel boundary and a perpendicular crossing through $(-2, 3)$. Draw both new lines on the supplied diagram.



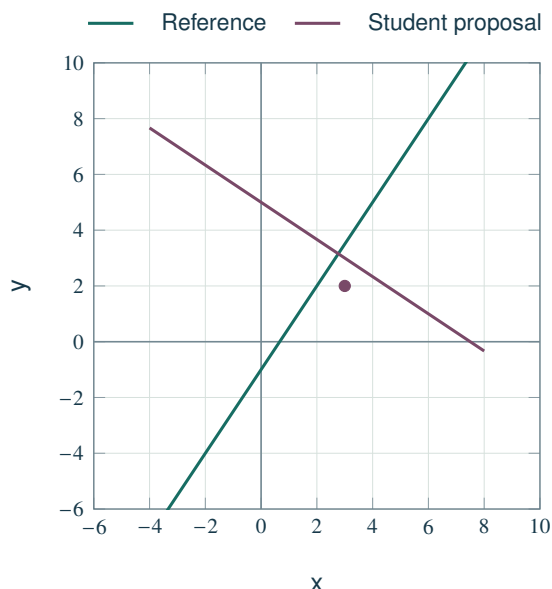
WORKING / DECISION

The existing vertical boundary and required point are supplied.

3. Check the engineering

Use the graph to record your thinking. Show calculations and include units where needed.

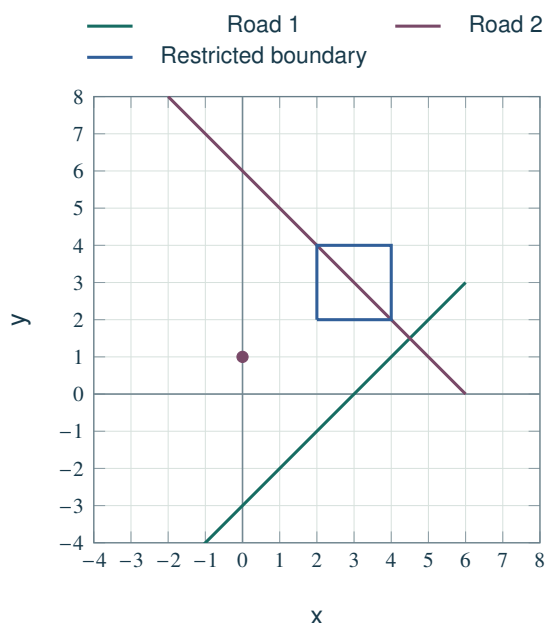
9. A student proposes $y = -\frac{2}{3}x + 5$ as a perpendicular to $y = \frac{3}{2}x - 1$ through $(3, 2)$. Check the gradient and point separately. Repair the equation if needed and show the repaired line on the graph.



WORKING / DECISION

The reference, the student proposal and the required point are supplied. The repaired line is not drawn.

10. A route through $(0, 1)$ may be parallel to either road $y = x - 3$ or road $y = -x + 6$. The entire square $2 \leq x \leq 4$, $2 \leq y \leq 4$ is restricted, including its boundary. Which proposed route avoids the square? Draw your route.



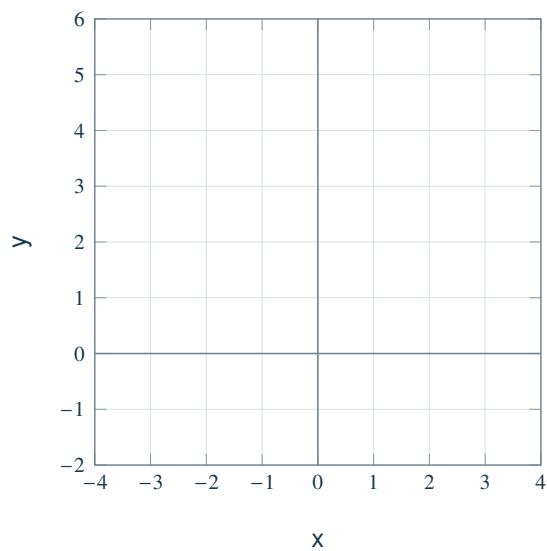
WORKING / DECISION

The closed square is restricted, including its edge. Existing roads are shown; construct the candidate routes through the marked point.

3. Check the engineering

Use the graph to record your thinking. Show calculations and include units where needed.

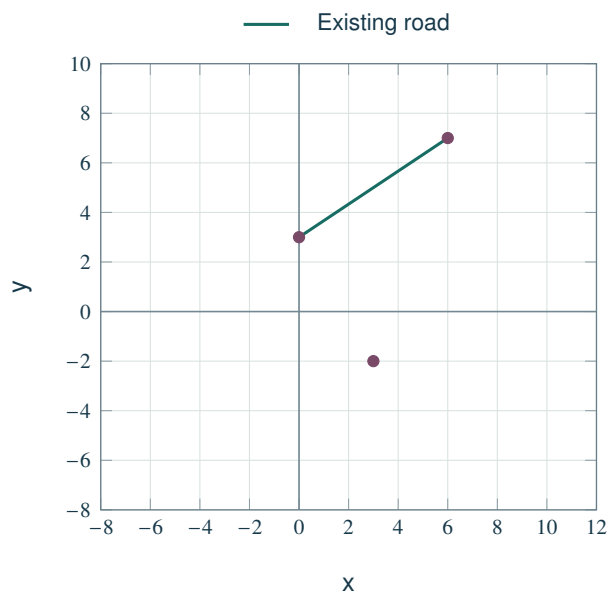
11. Sketch $y = x + 2$ and $y = -x + 2$ on equal-scale axes. Find their intersection and verify their angle using gradients.



Construct both lines and label their intersection.

WORKING / DECISION

12. A road passes through $(0, 3)$ and $(6, 7)$. Build a parallel service line through $(3, -2)$, then a perpendicular rescue line through the same point. Give both equations and one check for each, then draw the two new lines.



The existing road and the shared required point. Add both service and rescue routes.

WORKING / DECISION

6G / Worked answers

Teacher appendix. Equivalent exact forms are accepted. Use the reasoning to check student work.

1. (a) Parallel directions; (b) perpendicular; (c) neither.

- Equal gradients give parallel directions; distinct lines also need different intercepts.
 - $\left(-\frac{2}{7}\right)\left(\frac{7}{2}\right) = -1$; $4(-4) = -16$.
-

2. Yes: $m_A = \frac{1}{2}$ and $m_B = -2$.

- $m_A = \frac{4-1}{4-(-2)} = \frac{3}{6}$.
 - $m_B = \frac{2-6}{3-1} = -2$; their product is -1 .
-

3. (a) $-\frac{8}{5}$; (b) $\frac{1}{3}$; (c) 4.

- Change the sign and take the reciprocal.
 - $\frac{5}{8}\left(-\frac{8}{5}\right) = -1$, $(-3)\frac{1}{3} = -1$, $\left(-\frac{1}{4}\right)4 = -1$.
-

4. Parallel: A,C. Perpendicular: A,B and B,C.

- The gradients are $2, -\frac{1}{2}, 2, \frac{1}{2}$.
 - Only the pairs listed have equal gradients or product -1 .
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5. $y = -3x + 11$.

- $5 = -3(2) + c$, so $c = 11$.
 - At $x = 2$, $-6 + 11 = 5$.
-

6. $y = -\frac{5}{2}x + 7$.

- Required gradient is $-\frac{5}{2}$.
 - $-3 = -\frac{5}{2}(4) + c = -10 + c$, so $c = 7$.
-

6G / Worked answers

Teacher appendix. Equivalent exact forms are accepted. Use the reasoning to check student work.

7. $x = -3$, a vertical line with undefined gradient.

- A right angle to a horizontal line is vertical.
 - Its horizontal run is zero, so $\frac{\text{rise}}{\text{run}}$ is undefined.
-

8. Parallel: $x = -2$. Perpendicular: $y = 3$.

- A parallel vertical line keeps x constant at -2 .
 - The perpendicular line is horizontal with y always 3.
-

9. Gradient is correct; intercept should be 4. $y = -\frac{2}{3}x + 4$.

- $\frac{3}{2} \left(-\frac{2}{3}\right) = -1$.
 - The proposed line gives $y = 3$ at $x = 3$. Use $2 = -2 + c$ to get $c = 4$.
-

10. Choose $y = -x + 1$.

- The alternatives through $(0, 1)$ are $y = x + 1$ and $y = -x + 1$.
 - $y = x + 1$ enters the square; for $2 \leq x \leq 4$, $-x + 1$ is between -3 and -1 , below it.
-

11. Intersection $(0, 2)$; the lines are perpendicular.

- $x + 2 = -x + 2$ gives $2x = 0$.
 - Gradients 1 and -1 have product -1 .
-

12. Service: $y = \frac{2}{3}x - 4$. Rescue: $y = -\frac{3}{2}x + \frac{5}{2}$.

- Road gradient $= \frac{4}{6} = \frac{2}{3}$; $-2 = 2 + c$ gives service $c = -4$.
 - Perpendicular $m = -\frac{3}{2}$; $-2 = -\frac{9}{2} + c$ gives $c = \frac{5}{2}$. Both give $y = -2$ at $x = 3$.
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