

CHAPTER 6 / 6F

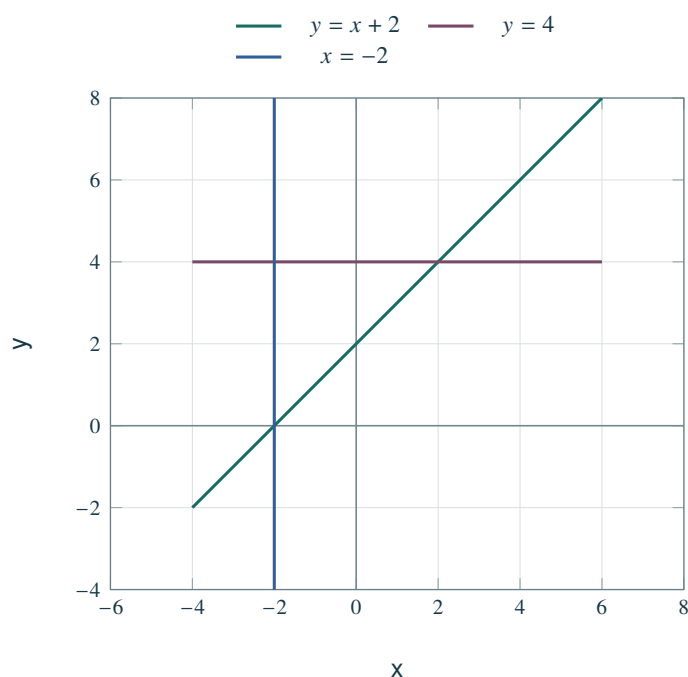
 $y = mx + c$ and special lines

Name: _____ Class: _____ Date: _____

Use the gradient and intercept to describe a line, sketch it and decide whether it fits a practical task. Distinguish sloping, horizontal and vertical lines.

What you will learn

- Identify m and c , including negative and fractional values.
- Build a line through a given point and check it by substitution.
- Recognise horizontal and vertical lines and their special equations.

See the mathematics

Compare a sloping line with horizontal and vertical special lines.

Key ideas

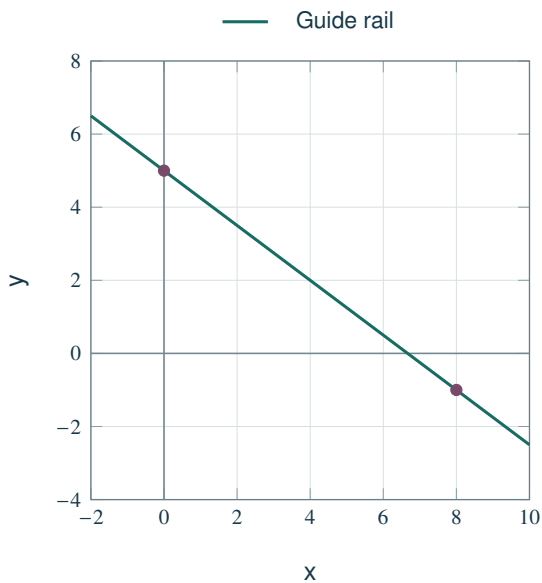
- In $y = mx + c$, m is the gradient and $(0, c)$ is the y -intercept.
- For $m = \frac{p}{q}$, a move of q units right changes y by p units. Keep the sign.
- A horizontal line has equation $y = k$ and gradient 0. A vertical line has equation $x = k$ and undefined gradient.
- A point lies on a line only if substituting both coordinates makes the equation true.

Pages 1–8: student resource. Pages 9–10: worked answers.

Worked examples / connect the calculation to the graph

1. Read a fractional gradient

A guide rail has equation $y = -\frac{3}{4}x + 5$. Identify the intercept and find the height at $x = 8$.



1. Compare with $y = mx + c$: $m = -\frac{3}{4}$ and $c = 5$.

2. Substitute $x = 8$: $y = -\frac{3}{4}(8) + 5 = -6 + 5$.

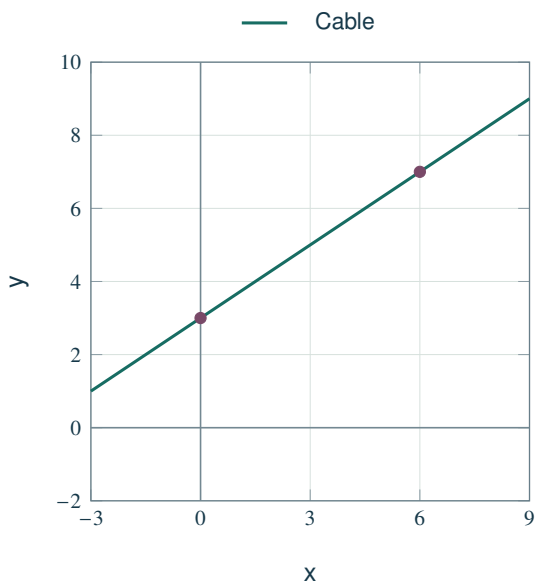
Result

The y -intercept is $(0, 5)$; the point at $x = 8$ is $(8, -1)$.

The worked model marks its intercept and the point found by substitution.

2. Build a line through a point

A cable has gradient $\frac{2}{3}$ and passes through $(6, 7)$. Find its equation.



1. Write $y = \frac{2}{3}x + c$.

2. Substitute $(6, 7)$: $7 = 4 + c$, so $c = 3$.

Result

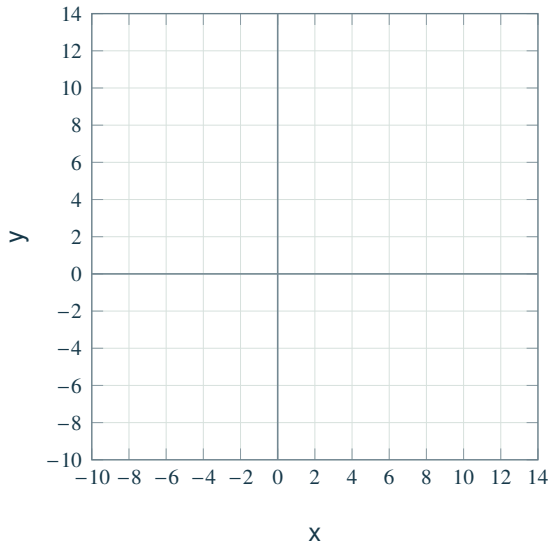
$y = \frac{2}{3}x + 3$.

The completed cable model passes through the required point and its calculated intercept.

1. Read and sketch

Use the graph to record your thinking. Show calculations and include units where needed.

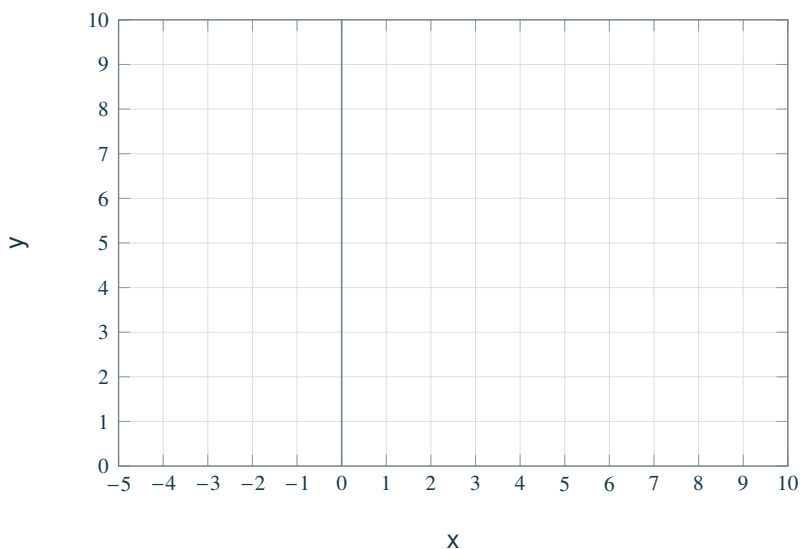
1. For $y = 4x - 7$, state the gradient and y -intercept. Find y when $x = 3$. Explain what a one-unit increase in x does to y . Use the grid to show the line and the point you calculate.



Construction grid for the line, intercept and calculated point.

WORKING / DECISION

2. For $y = -\frac{2}{5}x + 6$, state m and c . Starting at the y -intercept, give a second point reached by moving 5 units right. Mark both points on the grid and draw the line.



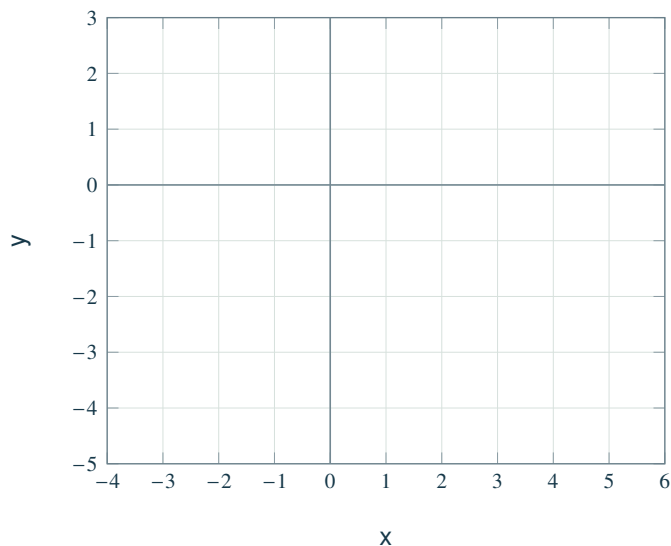
Construction grid for a fractional negative gradient.

WORKING / DECISION

1. Read and sketch

Use the graph to record your thinking. Show calculations and include units where needed.

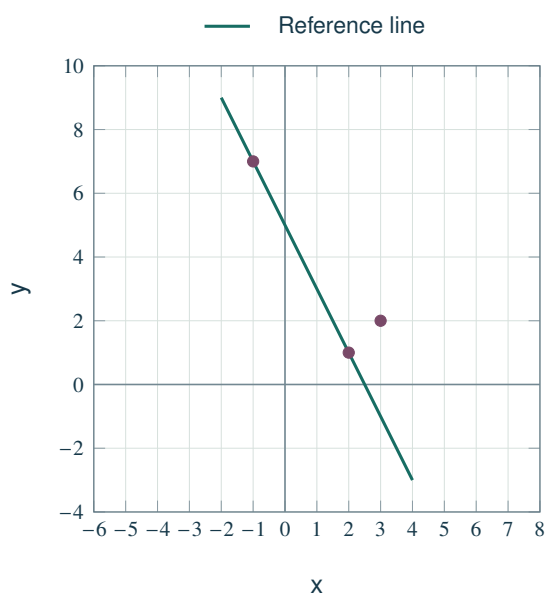
3. Sketch $y = \frac{1}{2}x - 2$ for $-4 \leq x \leq 6$. Mark the y -intercept and the point where the line crosses the x -axis.



WORKING / DECISION

Calculate the points before constructing the requested line segment.

4. Decide whether each point is on $y = -2x + 5$: $A(2, 1)$, $B(-1, 7)$, $C(3, 2)$. Show one substitution for each. Use the reference graph to check your decisions.



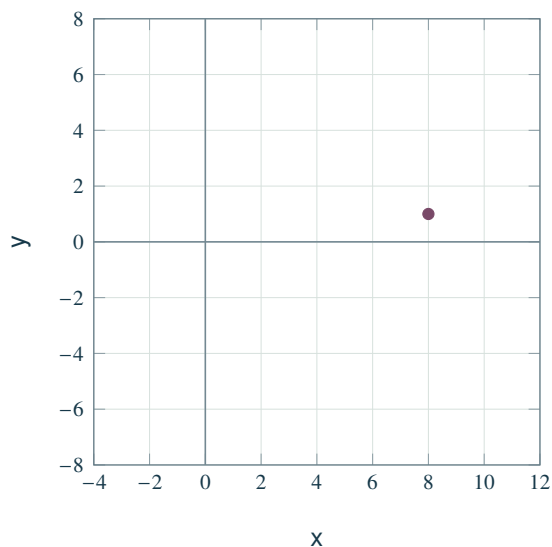
WORKING / DECISION

The supplied line and candidate points $A(2, 1)$, $B(-1, 7)$, $C(3, 2)$.

2. Design the equation

Use the graph to record your thinking. Show calculations and include units where needed.

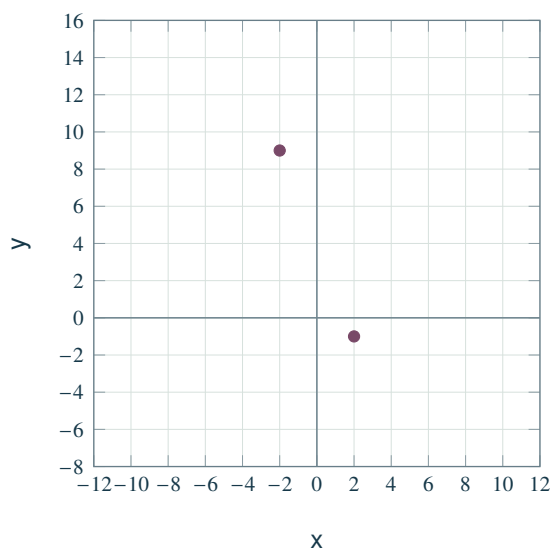
5. A survey line has gradient $\frac{3}{4}$ and must pass through $(8, 1)$. Find its equation and its y -intercept. Draw your proposed line through the marked point.



WORKING / DECISION

Only the required point is provided. Construct the survey line after finding its equation.

6. A ramp line has gradient $-\frac{5}{2}$ and passes through $(-2, 9)$. Find its equation. Does it pass through $(2, -1)$? Draw the line to support your check.



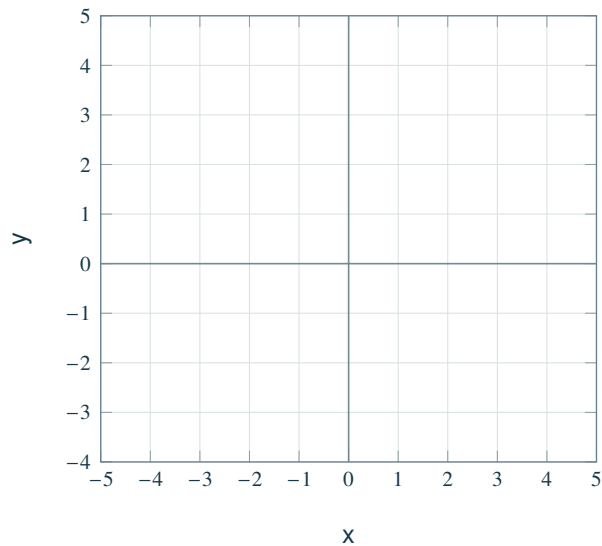
WORKING / DECISION

The required point and candidate check point are marked; the ramp remains for you to draw.

2. Design the equation

Use the graph to record your thinking. Show calculations and include units where needed.

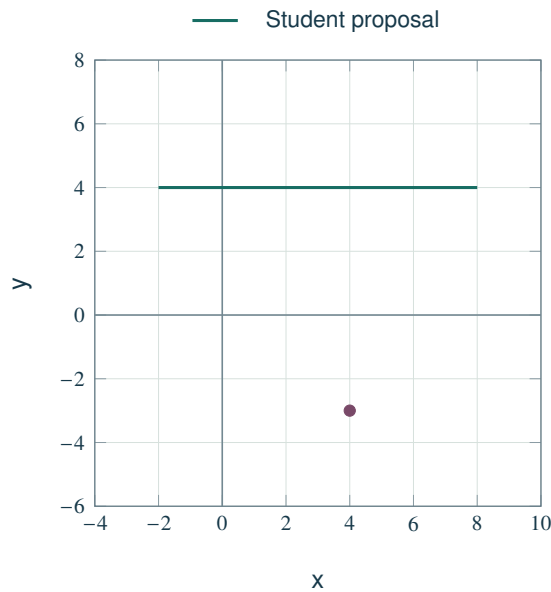
7. Draw $y = 3$ and $x = -2$ on the same grid. State each gradient and their intersection point.



Construction grid for both special lines.

WORKING / DECISION

8. A student says the vertical line through $(4, -3)$ is $y = 4$. Correct the equation, explain the mistake and state whether the line has a y -intercept. Draw the corrected line on the supplied diagram.



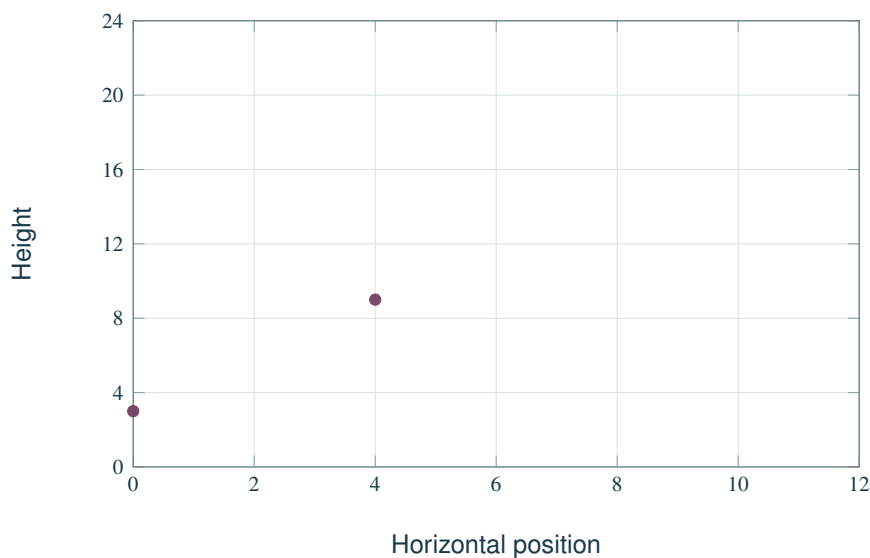
WORKING / DECISION

The student proposal and required point are given. Add your corrected line.

3. Apply and justify

Use the graph to record your thinking. Show calculations and include units where needed.

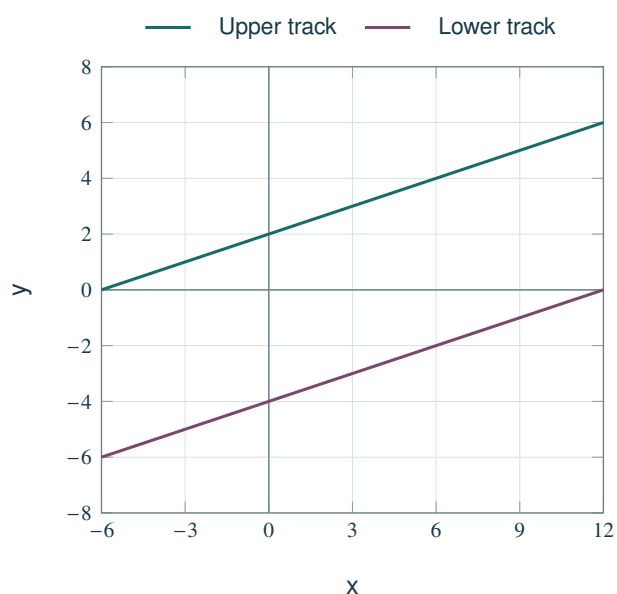
9. A sensor must show height 9 at horizontal position 4 and have a starting height of 3 at position 0. Find its linear rule and predict the height at position 10. Use the grid to show the two known readings and your predicted point.



WORKING / DECISION

The two known sensor readings are plotted. Add the recovered model and prediction.

10. Two tracks are $y = \frac{1}{3}x + 2$ and $y = \frac{1}{3}x - 4$. Compare their directions and find their vertical separation at $x = 9$. Is this the shortest distance between the tracks? Mark the vertical separation on the graph.



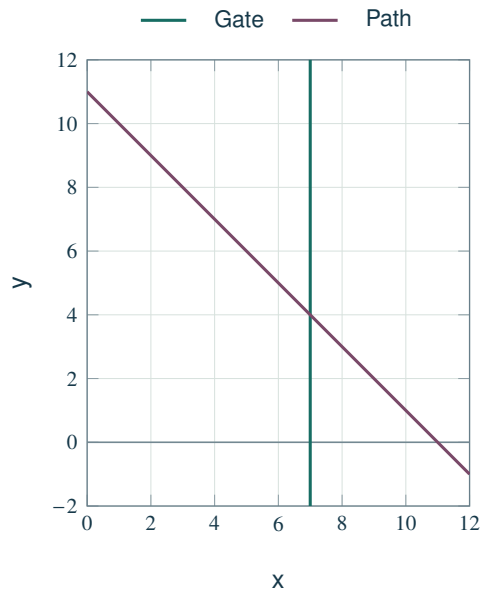
WORKING / DECISION

Both supplied tracks are shown. Mark the separation at the specified horizontal position.

3. Apply and justify

Use the graph to record your thinking. Show calculations and include units where needed.

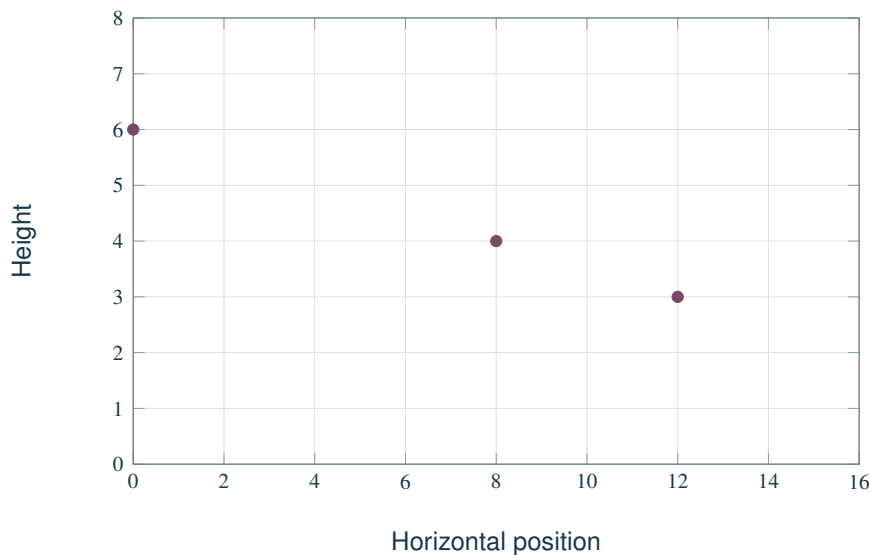
11. A gate is fixed at $x = 7$. A sloping path follows $y = -x + 11$. Find where the path meets the gate. State the gate's equation and why $y = mx + c$ cannot describe it. Mark the intersection on the graph.



WORKING / DECISION

The supplied gate and path; calculate and mark their meeting point.

12. A roof edge must pass through $(0, 6)$ and $(8, 4)$. A support at $x = 12$ reaches height 3. Does this single straight roof edge meet the support? Find and use its equation, then draw the roof on the diagram.



WORKING / DECISION

Two roof points and the top of the proposed support are supplied. Draw your roof model.

6F / Worked answers

Teacher appendix. Equivalent exact forms are accepted. Use the reasoning to check student work.

1. $m = 4$; intercept $(0, -7)$; $y = 5$; y rises by 4.

- m is the coefficient of x ; c is the constant term.
 - At $x = 3$, $y = 12 - 7 = 5$.
-

2. $m = -\frac{2}{5}$, $c = 6$; second point $(5, 4)$.

- A run of 5 produces a rise of -2 .
 - Start at $(0, 6)$, then move to $(5, 4)$.
-

3. Intercepts $(0, -2)$ and $(4, 0)$.

- At $x = -4$, $y = -4$; at $x = 6$, $y = 1$.
 - Draw a straight line through these points and both intercepts.
-

4. A and B lie on the line; C does not.

- A: $-2(2) + 5 = 1$. B: $-2(-1) + 5 = 7$.
 - C: $-2(3) + 5 = -1$, not 2.
-

5. $y = \frac{3}{4}x - 5$; y -intercept $(0, -5)$.

- $1 = \frac{3}{4}(8) + c = 6 + c$.
 - Therefore $c = -5$.
-

6. $y = -\frac{5}{2}x + 4$; yes.

- $9 = -\frac{5}{2}(-2) + c = 5 + c$, so $c = 4$.
 - At $x = 2$, $y = -5 + 4 = -1$.
-

6F / Worked answers

Teacher appendix. Equivalent exact forms are accepted. Use the reasoning to check student work.

7. Gradients 0 and undefined; intersection $(-2, 3)$.

- $y = 3$ is horizontal. $x = -2$ is vertical.
 - Their intersection satisfies both equations.
-

8. $x = 4$; no y -intercept.

- Every point on a vertical line has the same x -coordinate.
 - $y = 4$ would be horizontal. The line $x = 4$ never reaches $x = 0$.
-

9. $y = \frac{3}{2}x + 3$; height 18.

- $m = \frac{9-3}{4-0} = \frac{6}{4} = \frac{3}{2}$.
 - At $x = 10$, $y = 15 + 3 = 18$.
-

10. Same gradient; vertical separation 6; not the shortest distance.

- At $x = 9$ the heights are 5 and -1 , a vertical difference of 6.
 - The shortest distance between sloping parallel lines is measured perpendicular to them.
-

11. Intersection $(7, 4)$; gate $x = 7$.

- Set $x = 7$: $y = -7 + 11 = 4$.
 - A vertical line has multiple y -values for the same x and undefined gradient.
-

12. Yes; $y = -\frac{1}{4}x + 6$ gives $y = 3$ at $x = 12$.

- $m = \frac{4-6}{8-0} = -\frac{1}{4}$, with $c = 6$.
 - Substitution gives $-\frac{1}{4}(12) + 6 = 3$.
-