

CHAPTER 6 / 6D

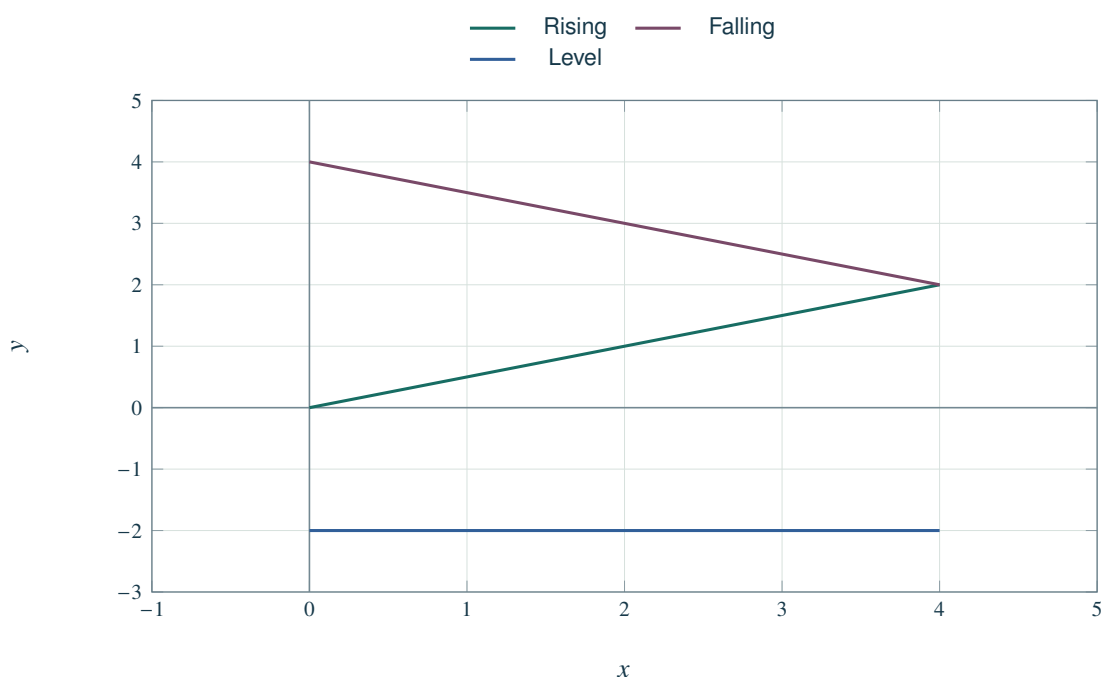
Exploring gradient

Name: _____ Class: _____ Date: _____

Measure steepness with signed rise and run. Compare slopes on equal scales, distinguish horizontal and vertical lines, and use gradient in practical design decisions.

What you will learn

- Calculate gradient from graphs or two points.
- Interpret positive, negative, zero and undefined gradients.
- Express gradients as fractions, decimals and percentages in context.

See the mathematics

Learning map: compare the signed height change for the same horizontal run.

Key ideas

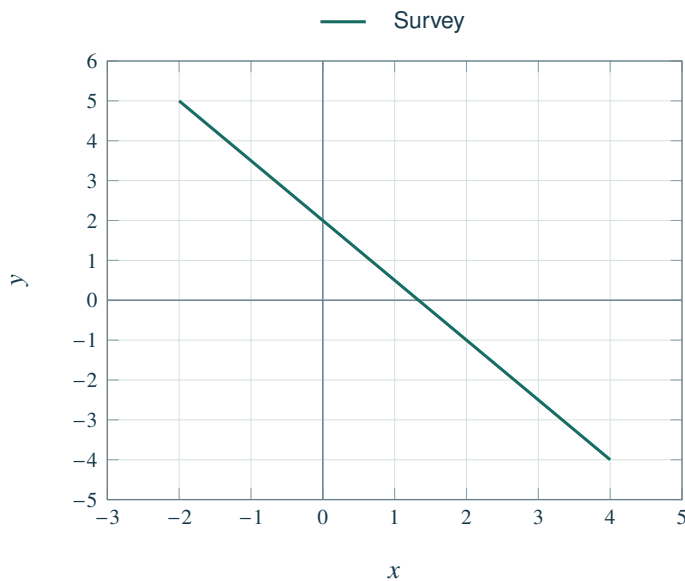
- Gradient is $m = \frac{y_2 - y_1}{x_2 - x_1}$. Keep the same point order in both differences.
- A positive gradient rises from left to right; a negative gradient falls.
- A horizontal line has $m = 0$; a vertical line has undefined gradient.
- Percentage gradient is $100m\%$. Compare $|m|$ when judging steepness.

Pages 1–8: student resource. Pages 9–10: worked answers.

Worked examples / connect the calculation to the graph

1. A signed gradient

Find the gradient through $A(-2, 5)$ and $B(4, -4)$.



1. Signed rise = $-4 - 5 = -9$; run = $4 - (-2) = 6$.

2. $m = \frac{-9}{6} = \frac{-3}{2}$.

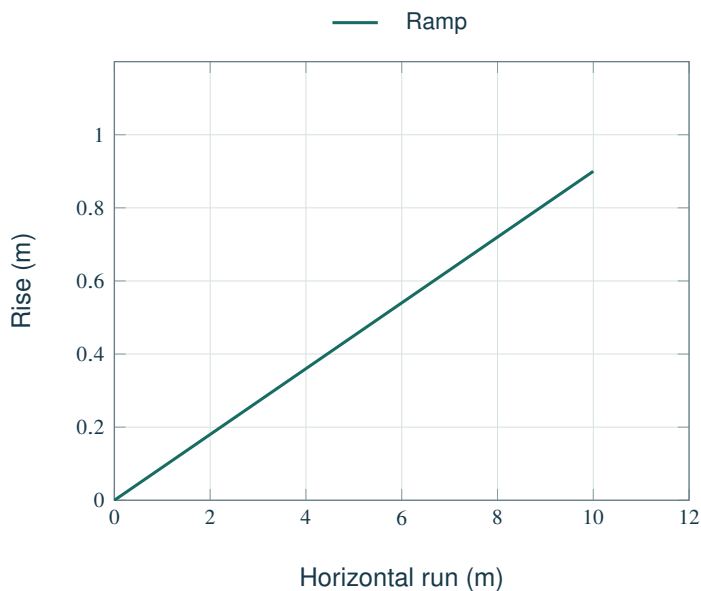
Result

$m = \frac{-3}{2}$. The line falls left to right.

Worked example: preserve the same point order in both coordinate changes.

2. Design a gentle ramp

A ramp rises 0.9 m over a 10 m horizontal run. Express its gradient as a decimal and percentage.



1. Gradient = rise/run = $\frac{0.9}{10} = 0.09$.

2. Multiply by 100 to convert to percentage gradient.

Result

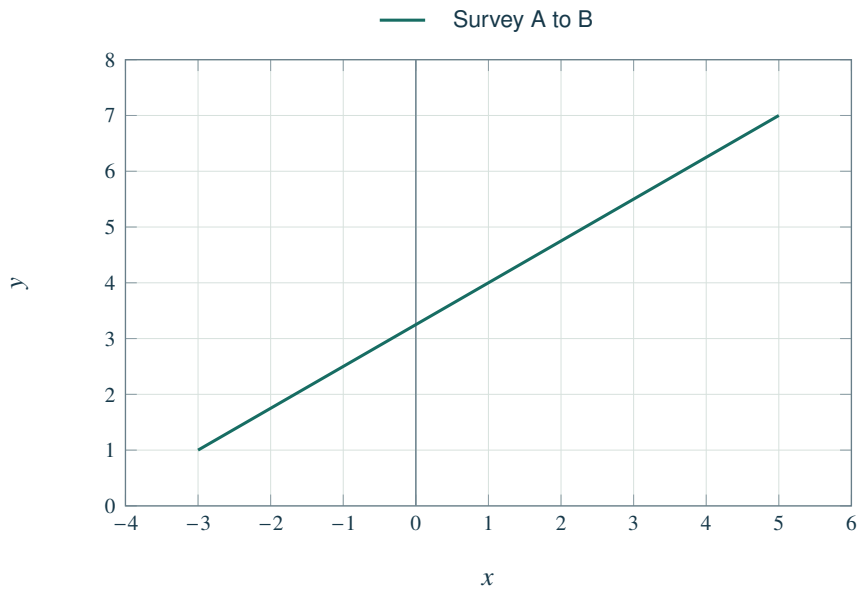
Gradient $0.09 = 9\%$.

Worked example: express one rise-to-run ratio as a decimal and percentage.

Signed rise and run

Use the graph to record your thinking. Show calculations and include units where needed.

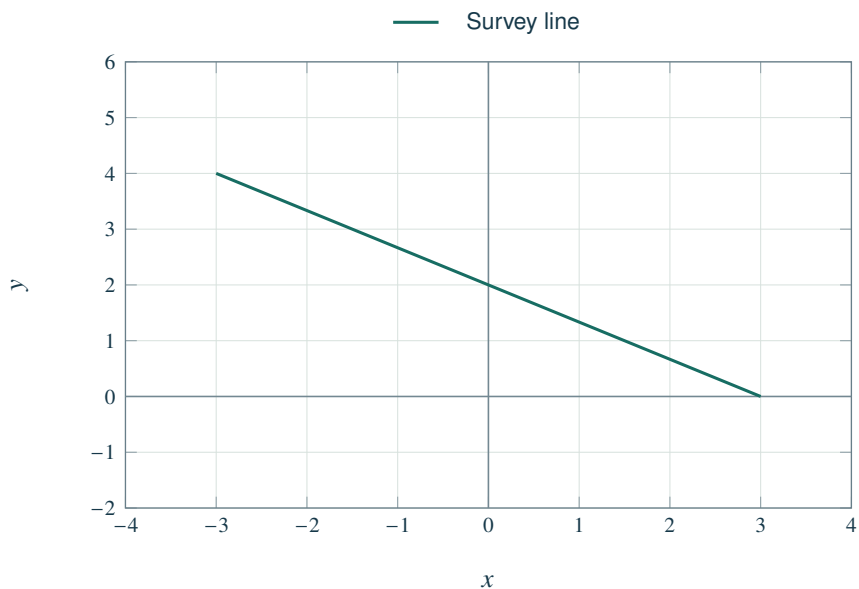
1. Find the signed rise, run and gradient from $A(-3, 1)$ to $B(5, 7)$.



Use the same two points for both signed coordinate changes.

WORKING / DECISION

2. Find the gradient of the line shown. State whether it rises or falls from left to right.



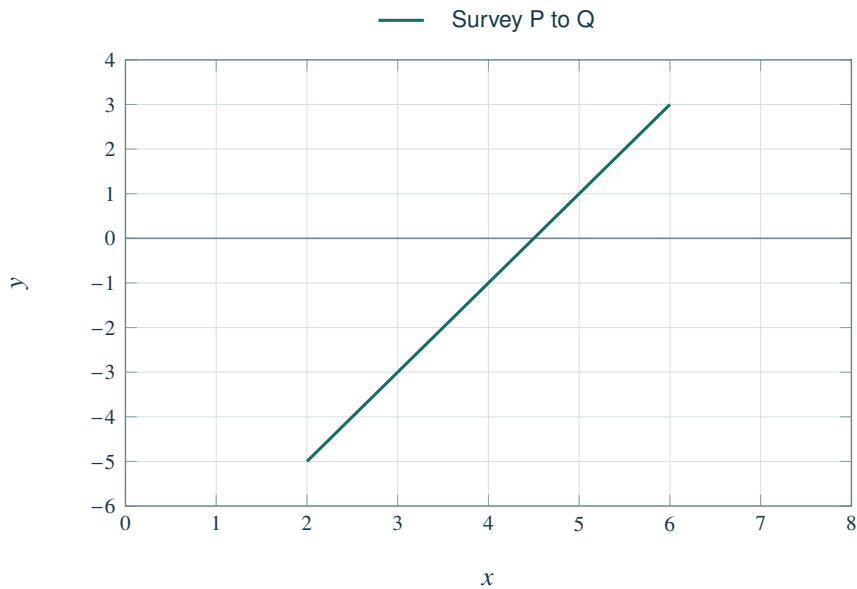
Read the quantities, scales and labelled line before answering.

WORKING / DECISION

Signed rise and run

Use the graph to record your thinking. Show calculations and include units where needed.

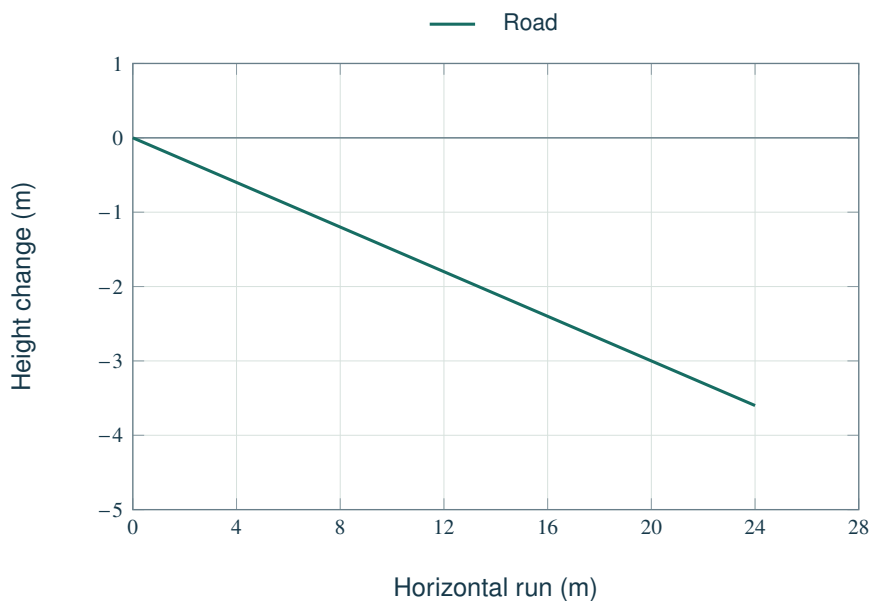
3. Two survey points are $P(2, -5)$ and $Q(6, 3)$. Calculate the gradient using P first, then Q first. Explain why the result is unchanged.



WORKING / DECISION

Calculate in each direction along the same line.

4. A road falls 3.6 m while moving 24 m horizontally to the right. Give its signed gradient as a fraction and decimal.



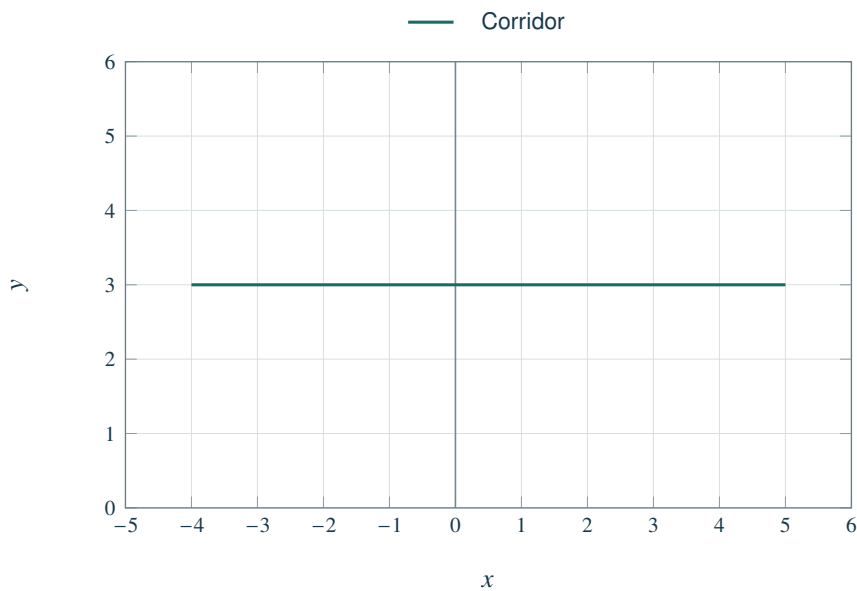
WORKING / DECISION

The vertical change is measured relative to the starting road height.

Special lines and comparisons

Use the graph to record your thinking. Show calculations and include units where needed.

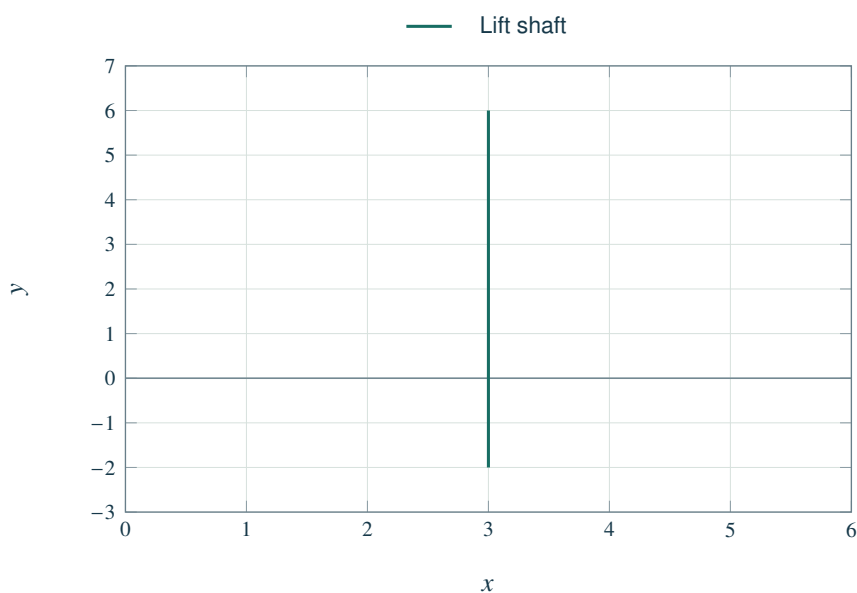
5. Find the gradient through $(-4, 3)$ and $(5, 3)$. Explain the result using rise and run.



WORKING / DECISION

Interpret the horizontal and vertical changes separately.

6. A lift shaft joins $(3, -2)$ and $(3, 6)$. State its gradient and explain why it is not zero.



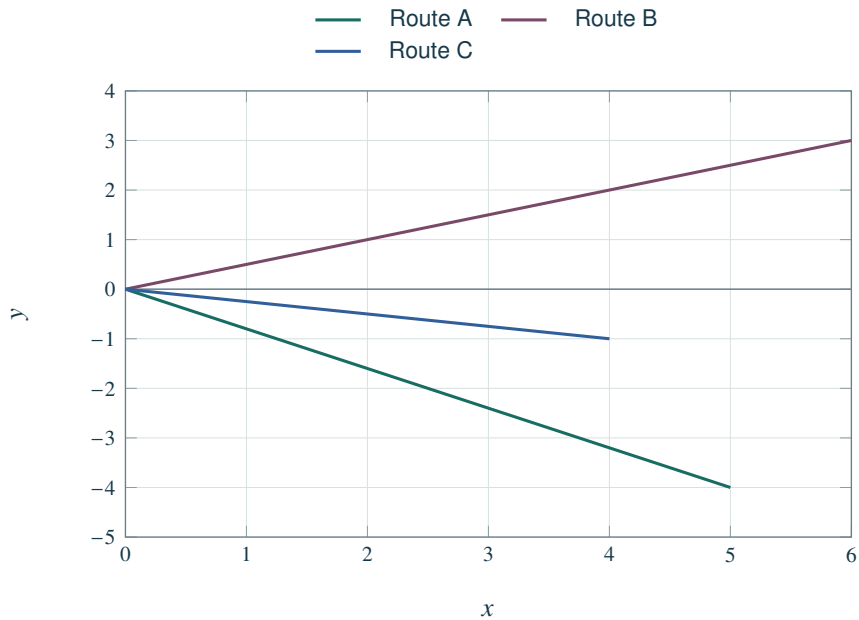
WORKING / DECISION

The shaft endpoints share one coordinate. Use that information to describe its gradient.

Special lines and comparisons

Use the graph to record your thinking. Show calculations and include units where needed.

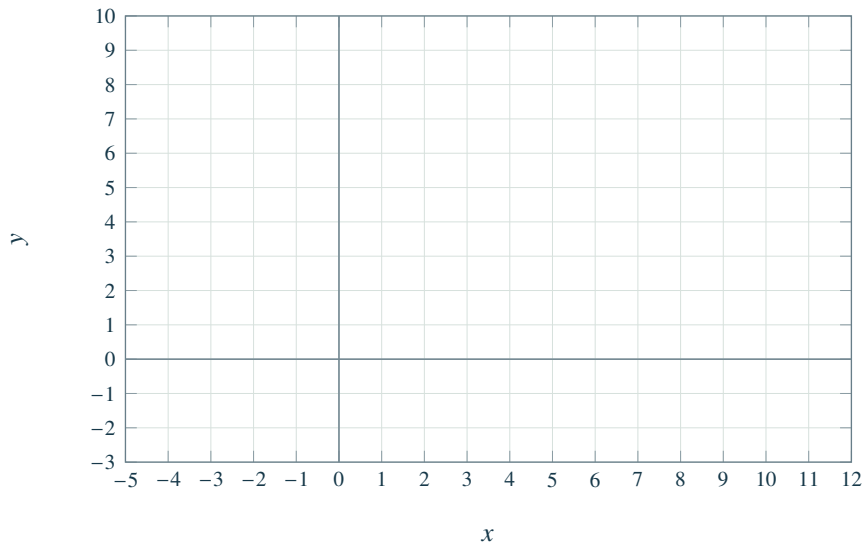
7. Routes have gradients $A = \frac{-4}{5}$, $B = \frac{1}{2}$ and $C = \frac{-1}{4}$. Order them from gentlest to steepest.



WORKING / DECISION

All three routes use the same scales so their steepness can be compared.

8. A line has gradient $\frac{3}{5}$ and passes through $(1, 2)$. Find the point on the line 10 units to the right. Find the point 5 units to the left.



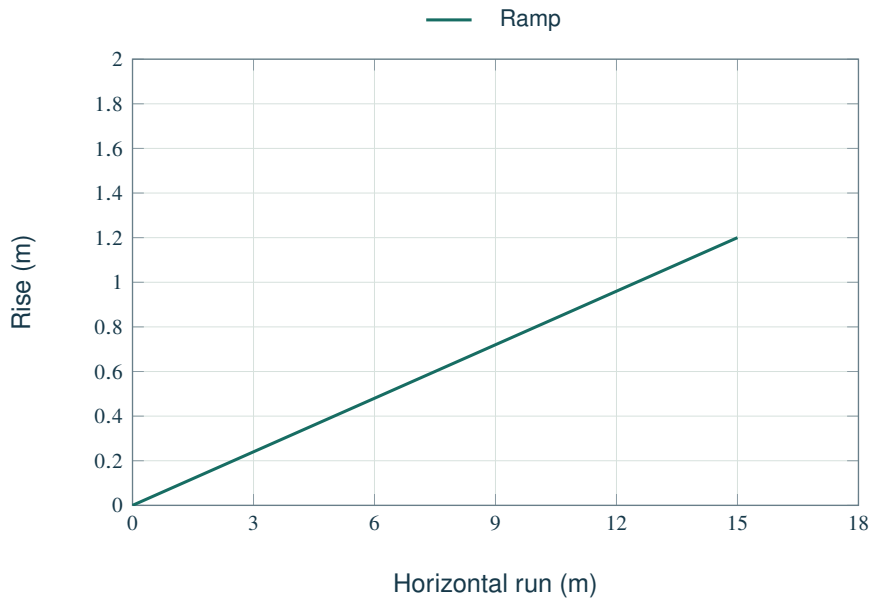
WORKING / DECISION

Start at the given point and construct both requested moves.

Engineering decisions

Use the graph to record your thinking. Show calculations and include units where needed.

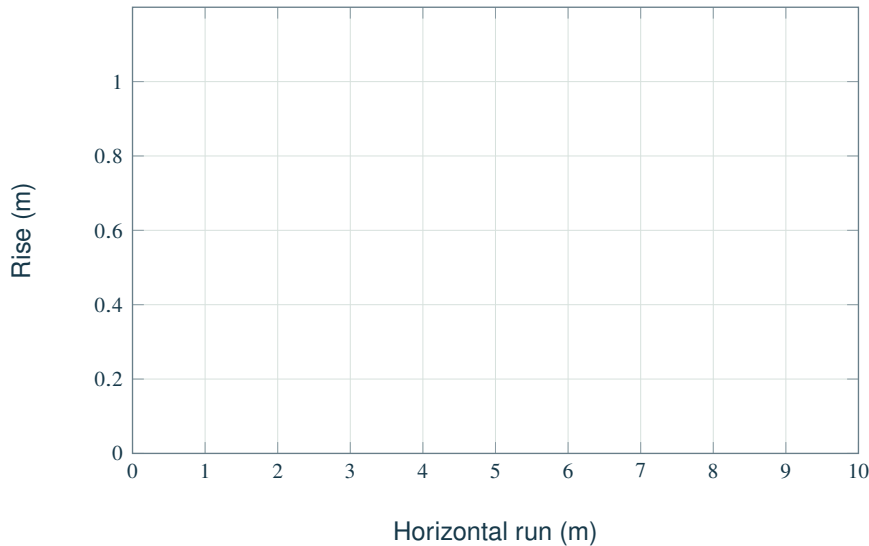
9. A ramp rises 1.2 m over a 15 m run. Express its gradient as a fraction, decimal and percentage.



WORKING / DECISION

Read the rise and run of the ramp; express the same slope in three forms.

10. For this puzzle, a service ramp may have gradient no greater than $\frac{1}{10}$. It rises 0.8 m. Find its minimum horizontal run. Does it fit within 9 m?



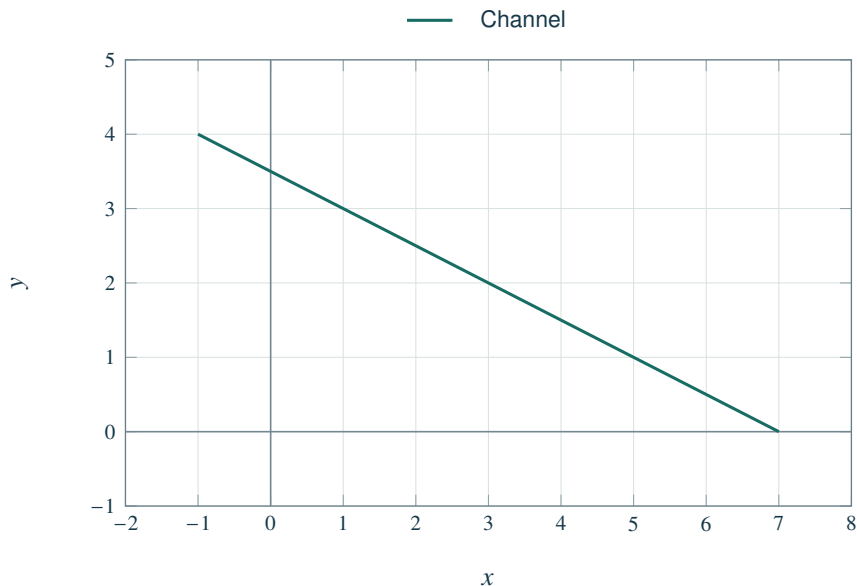
WORKING / DECISION

Draw your proposed ramp to the 0.8 m platform and check it against the stated design limit.

Engineering decisions

Use the graph to record your thinking. Show calculations and include units where needed.

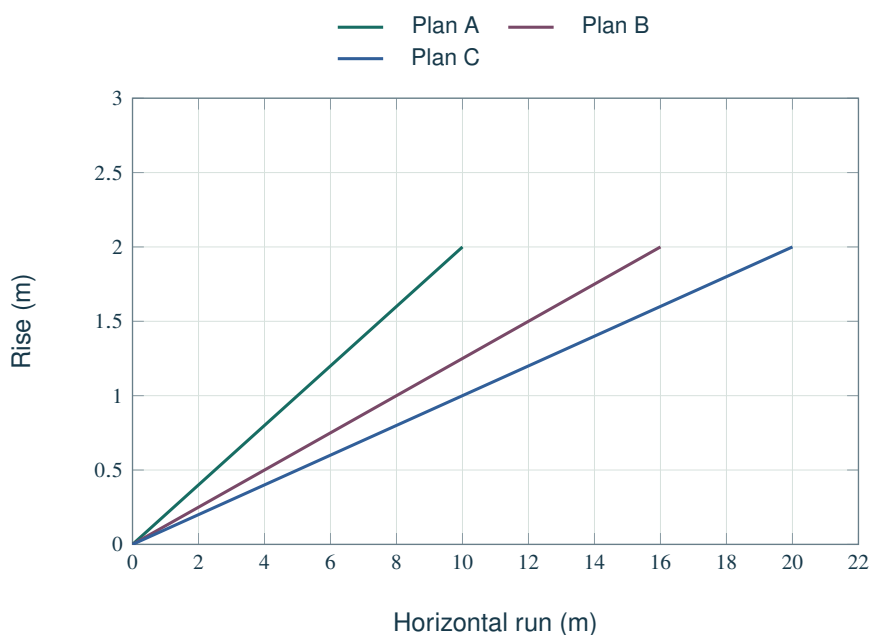
- 11.** A channel runs from $(-1, 4)$ to $(7, 0)$. A surveyor reports gradient $+\frac{1}{2}$. Give the correct gradient and explain the likely error.



WORKING / DECISION

Compare the surveyor's stated sign with the actual direction from left to right.

- 12.** A bridge rises 2 m. Plans have runs A: 10m, B: 16m, C: 20 m. The gradient must be at most $\frac{1}{8}$. Each metre of run costs \$12 and the budget is \$210. Choose a plan and give its gradient, percentage and cost.



WORKING / DECISION

Compare the profiles, then use the price per metre and budget from the question.

6D / Worked answers

Teacher appendix. Equivalent exact forms are accepted. Use the reasoning to check student work.

1. Rise 6; run 8; gradient $\frac{3}{4}$.

- Rise = $7 - 1 = 6$; run = $5 - (-3) = 8$.
 - $m = \frac{6}{8} = \frac{3}{4}$.
-

2. $m = \frac{-2}{3}$; it falls.

- Using $(-3, 4)$ and $(3, 0)$, rise = $0 - 4 = -4$ and run = 6.
 - $m = \frac{-4}{6} = \frac{-2}{3}$.
-

3. Both calculations give $m = 2$.

- P to Q: $(3 - (-5))/(6 - 2) = \frac{8}{4} = 2$.
 - Q to P: $\frac{-5-3}{2-6} = \frac{-8}{-4} = 2$; both signs reverse.
-

4. $\frac{-3}{20} = -0.15$.

- Rise is $-3.6m$ because the road falls.
 - $m = \frac{-3.6}{24} = -0.15 = \frac{-3}{20}$.
-

5. 0; it is a horizontal line.

- Rise = $3 - 3 = 0$; run = $5 - (-4) = 9$.
 - $m = \frac{0}{9} = 0$.
-

6. Undefined; the run is zero.

- Rise = 8, while run = $3 - 3 = 0$.
 - The quotient $\frac{8}{0}$ is undefined; zero gradient would require zero rise instead.
-

6D / Worked answers

Teacher appendix. Equivalent exact forms are accepted. Use the reasoning to check student work.

7. C, B, A.

- Compare absolute values: $\frac{1}{4} = 0.25$, $\frac{1}{2} = 0.5$, $\frac{4}{5} = 0.8$.
 - The negative signs describe falling direction, not reduced steepness.
-

8. (11, 8); (−4, −1).

- For a run of 10, rise = $(\frac{3}{5}) \times 10 = 6$.
 - For a run of −5, rise = $(\frac{3}{5}) \times (-5) = -3$.
-

9. $\frac{2}{25} = 0.08 = 8\%$.

- $m = \frac{1.2}{15} = 0.08$.
 - $0.08 \times 100 = 8\%$.
-

10. Minimum run 8 m; yes.

- $0.8/\text{run} \leq \frac{1}{10}$ implies run ≥ 8 .
 - An 8 m run fits within the 9 m space.
-

11. $-\frac{1}{2}$; the surveyor ignored the falling direction or mixed point orders.

- Rise = $0 - 4 = -4$ and run = $7 - (-1) = 8$.
 - $m = \frac{-4}{8} = -\frac{1}{2}$.
-

12. Plan B; gradient $\frac{1}{8} = 12.5\%$; cost \$192.

- A has gradient $\frac{2}{10} = \frac{1}{5}$, which is too steep.
 - B meets the limit and costs $16 \times 12 = \$192$; C costs \$240, above budget.
-